# Parameter Identification in Discrete-choice Models

## TheoryGuru applications



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## Background

Here we consider a discrete choice that depends on observed factors $x$ and $y$ as well as unobserved factors summarized by $\sigma\,\epsilon$, were $\epsilon$ is normalized to have unit variance and distribution function $F$. The discrete choice is made if and only if:

$$\sigma\,\epsilon < \beta\,x + \gamma\,y$$

The log likelihood function for observing $\{x_1, y_1, d_1, x_2, y_2, d_2, \ldots, x_n, y_n, d_n\}$ is:

$$L\left[\left\{\frac{\beta}{\sigma}, \frac{\gamma}{\sigma}\right\}\right] == \sum_{i\,|\,d_1==0} \mathrm{Log}\left[1 - F\left[\frac{\beta}{\sigma} x_i + \frac{\gamma}{\sigma} y_i\right]\right] + \sum_{i\,|\,d_1==1} \mathrm{Log}\left[F\left[\frac{\beta}{\sigma} x_i + \frac{\gamma}{\sigma} y_i\right]\right]$$

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## Setup

```
Get["http://economicreasoning.com"]
```

**Proof & Logic Tools 6.2**



Type ERCommands for a list of commands in the package.

| | | |
|---|---|---|
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```
LikelihoodisStrictlyConcave = (Not@SameVector[{β1/σ1, γ1/σ1}, {β2/σ2, γ2/σ2}] ∧ 0 < λ < 1) ⇒
    (1 - λ) L[{β1/σ1, γ1/σ1}] + λ L[{β2/σ2, γ2/σ2}] < L[(1 - λ) {β1/σ1, γ1/σ1} + λ {β2/σ2, γ2/σ2}];
(* for any {β1/σ1, γ1/σ1} and {β2/σ2, γ2/σ2} and λ *)
Vector1MaxesLikelihood = L[{β1/σ1, γ1/σ1}] ≥ L[(1 - λ) {β1/σ1, γ1/σ1} + λ {β2/σ2, γ2/σ2}];
Vector2MaxesLikelihood = L[{β2/σ2, γ2/σ2}] ≥ L[(1 - λ) {β1/σ1, γ1/σ1} + λ {β2/σ2, γ2/σ2}];
```

# Results

## Coefficients not identified

```
TheoryGuru[{0 < λ < 1,
  LikelihoodisStrictlyConcave, Vector1MaxesLikelihood, Vector2MaxesLikelihood,
  SameVector[{β1/σ1, γ1/σ1}, {β2/σ2, γ2/σ2}] ⇒ (β1/σ1 == β2/σ2 ∧ γ1/σ1 == γ2/σ2)},

 β1 == β2 ∨ γ1 == γ2]
```

```
True for some, False for others
```

```
TheoryInstance[{0 < λ < 1,
  LikelihoodisStrictlyConcave, Vector1MaxesLikelihood, Vector2MaxesLikelihood,
  SameVector[{β1/σ1, γ1/σ1}, {β2/σ2, γ2/σ2}] ⇒ (β1/σ1 == β2/σ2 ∧ γ1/σ1 == γ2/σ2),
  σ1 > 0, σ2 > 0},

 Not[β1 == β2 ∨ γ1 == γ2]]
```

| SameVector[{β1/σ1, γ1/σ1}, {β2/σ2, γ2/σ2}] | λ | L[{β1/σ1, γ1/σ1}] | L[{((1 - λ) β1) / σ1 + λ β2/σ2, ((1 - λ) γ1) / σ1 + λ γ2/σ2}] | L[{β2/σ2, γ2/σ2}] | β1 | β2 | γ1 | γ2 | σ1 | σ2 |
|---|---|---|---|---|---|---|---|---|---|---|
| True | 1/2 | 0 | -1 | 0 | -1 | -1/2 | -2 | -1 | 1 | 1/2 |

## Coefficients identified up to scale

```
TheoryGuru[{0 < λ < 1,
  LikelihoodisStrictlyConcave, Vector1MaxesLikelihood, Vector2MaxesLikelihood},
```

$$\text{SameVector}\left[\left\{\frac{\beta_1}{\sigma_1}, \frac{\gamma_1}{\sigma_1}\right\}, \left\{\frac{\beta_2}{\sigma_2}, \frac{\gamma_2}{\sigma_2}\right\}\right]\Big]$$

```
True
```

```
TheoryGuru[{0 < λ < 1,
  LikelihoodisStrictlyConcave, Vector1MaxesLikelihood, Vector2MaxesLikelihood,
```

$$\text{SameVector}\left[\left\{\frac{\beta_1}{\sigma_1}, \frac{\gamma_1}{\sigma_1}\right\}, \left\{\frac{\beta_2}{\sigma_2}, \frac{\gamma_2}{\sigma_2}\right\}\right] \Rightarrow \left(\frac{\beta_1}{\sigma_1} == \frac{\beta_2}{\sigma_2} \wedge \frac{\gamma_1}{\sigma_1} == \frac{\gamma_2}{\sigma_2}\right)\Big\},$$

$$\frac{\beta_1}{\gamma_1} == \frac{\beta_2}{\gamma_2}\Big]$$

```
True
```

## Variable interpretations