The Ethanol-subsidy Multiplier

TheoryGuru applied to Chicago Price Theory

Setup

Load Economicreasoning package only if it is not already loaded

```
In[•]:= If[Length@Names["PLTools`*"] < 10,
          Get["http://economicreasoning.com"]]
```

Proof & Logic Tools 6.3

(c) Copyright 2016, 2017, 2018, 2019 by JMJ Economics

Type ERCommands for a list of commands in the package.

Introduction to Automated Economic Reasoning		
Tutorials:	Entering calculus	General Mathematica tips
Get started	Load extras	Browse examples

Essentials

x is the subsidy for corn used in ethanol

```
In[•]:= Equilibrium = eth[p - x] + feed[p] == s[p];

In[•]:= d[p_] = eth[p - x] + feed[p];

In[•]:= InvSupply = p == c'[s[p]];

In[•]:= signconditions = {eth'[p - x] < 0, feed'[p] ≤ 0, s'[p] ≥ 0,
            x ≥ 0, eth[p - x] > 0, feed[p] > 0, s[p] > 0};
```

Shorthand notation defined

```
In[•]:= subsidyamt = eth[p - x] x(* subsidized quantity × subsidy rate *);

In[•]:= farmersurplus = s[p] p - c[s[p]];

In[•]:= inelasticsupply = s'[p] == 0;
```

```
In[•]:= qweight = eth[p - x] / d[p];

In[•]:= elasweight = (eth'[p - x] / eth[p - x]) / (d'[p] / d[p]);

In[•]:= incidenceparam = d'[p] / (d'[p] - s'[p]);

In[•]:= elascond = feed'[p] / feed[p] - d'[p] / d[p] > (eth[p - x] / feed[p]) (s'[p] / s[p]);

In[•]:= elasconds0 = eth'[p - x] / eth[p - x] < feed'[p] / feed[p];

In[•]:= elasgap = feed'[p] / feed[p] - eth'[p - x] / eth[p - x];
```

Results in the neighborhood of no subsidy

Any supply slope

```
In[•]:= TheoryGuru[{dEquilibrium/dx, Equilibrium, InvSupply, signconditions, x == 0, elascond},
  dfarmersurplus/dx > dsubsidyamt/dx > 0]

Out[•]= True

In[•]:= TheoryOverlap[
  {dEquilibrium/dx, Equilibrium, InvSupply, signconditions, x == 0}, elascond,
  dfarmersurplus/dx > dsubsidyamt/dx]

Out[•]= { feed'(p)/feed(p) - (eth'(p-x)+feed'(p))/(eth(p-x)+feed(p)) > eth(p-x) s'(p)/(feed(p) s(p)),
    -s'(p) dp/dx c'(s(p)) + p s'(p) dp/dx + s(p) dp/dx >
      x (dp/dx - 1) eth'(p - x) + eth(p - x) }        are equivalent

In[•]:= TheoryGuru[{dEquilibrium/dx, Equilibrium, signconditions, x == 0, elascond},
  dp/dx == qweight elasweight incidenceparam]

Out[•]= True
```

```
In[•]:= TheoryGuru[
          {∂Equilibrium/∂x, Equilibrium, InvSupply, signconditions, inelasticsupply, x == 0},
          ∂farmersurplus/∂x == s[p] ∂p/∂x &&
           ∂subsidyamt/∂x == s[p] qweight]
Out[•]= True
```

i.e., it is a comparison between pass-through rate and quantity weight, which amounts to a comparison of the elasticity weight and the incidence parameter

```
In[•]:= TheoryGuru[{∂Equilibrium/∂x, Equilibrium, InvSupply, signconditions, x == 0},
          SameSign[∂farmersurplus/∂x - ∂subsidyamt/∂x, ∂p/∂x - qweight] &&
           SameSign[∂farmersurplus/∂x - ∂subsidyamt/∂x, elasweight incidenceparam - 1]]
Out[•]= True
```

In order for farmers to benefit, ethanol demand must be more elastic

```
In[•]:= TheoryGuru[{∂Equilibrium/∂x, Equilibrium, InvSupply, signconditions, x == 0,
            ∂farmersurplus/∂x > ∂subsidyamt/∂x},
          elasgap > 0]
Out[•]= True
```

With horizontal ethanol demand, farmers locally gain more than the treasury pays

In Figure I-2 of Chicago Price Theory, feed'[*p*] and *s*'[*p*] are finite, while *eth*'[*p*] == −∞, so that $\frac{dp}{dx}$ == 1 and elascond = True (in the neighborhood of x == 0).

```
In[•]:= HorizontalEquilibrium = {eth + feed[p] == s[p], p == 1 + x};
```

```
In[•]:= HorizontalSubsidyamt = x eth;
```

```
In[•]:= TheoryGuru[{∂HorizontalEquilibrium/∂x, First@HorizontalEquilibrium,
          p == c'[s[p]], eth > 0, feed[p] > 0, (s'[p] == 0 ∧ feed'[p] == 0) ∨ x == 0},
         ∂farmersurplus/∂x > ∂HorizontalSubsidyamt/∂x > 0]
Out[•]= True
```

Note that the 1-for-1 pass-through rules out the case where the initial price is initially above the price that the ethanol-corn market will bear

Global results: farmers gain and Treasury pays

```
In[•]:= TheoryGuru[{∂Last@HorizontalEquilibrium/∂x,
          First@HorizontalEquilibrium, p == c'[s[p]], eth > 0, feed[p] > 0},
         ∂farmersurplus/∂x > 0]
Out[•]= True
```

```
In[•]:= TheoryGuru[{∂HorizontalEquilibrium/∂x, eth > 0, feed'[p] ≤ 0, s'[p] ≥ 0, x > 0},
         ∂HorizontalSubsidyamt/∂x > 0]
Out[•]= True
```

Not a global result without extra conditions

```
In[•]:= TheoryGuru[{∂HorizontalEquilibrium/∂x, HorizontalEquilibrium, p == c'[s[p]],
          eth > 0, x > 0, signconditions[[{2, 3, 4, 6, 7}]], feed'[p] < 0},
         ∂farmersurplus/∂x > ∂HorizontalSubsidyamt/∂x]
Out[•]= True for some, False for others
```

```
In[•]:= extracondition = x < feed[p]/(s'[p] - feed'[p]);
```

The old subsidy *x* has to be paid on the new supply *s*'[*p*] dx, but marginal supply gets no surplus. The old subsidy *x* has to be paid on quantity (–feed'[*p*] dx) reallocated from feed to ethanol, but this quantity was already getting paid by feed buyers and therefore yields no surplus to corn farmers. On the other hand, the extra subsidy *dx* tranlates to additional corn revenue from *feed* without a

subsidy expenditure.

```
In[•]:= TheoryOverlap[{∂HorizontalEquilibrium/∂x, HorizontalEquilibrium,
  p == c'[s[p]], eth > 0, x > 0, signconditions[[{2, 3, 4, 6, 7}]], feed'[p] < 0},
 extracondition,

 ∂farmersurplus/∂x > ∂HorizontalSubsidyamt/∂x]
```

Out[•]= $\left\{x < \frac{\text{feed}(p)}{s'(p) - \text{feed}'(p)},\ -s'(p)\ \frac{\partial p}{\partial x}\ c'(s(p)) + p\ s'(p)\ \frac{\partial p}{\partial x} + s(p)\ \frac{\partial p}{\partial x} > x\ \frac{\partial \text{eth}}{\partial x} + \text{eth}\right\}$ are equivalent

```
In[•]:= TheoryOverlap[{First@HorizontalEquilibrium, eth > 0, x > 0, feed'[p] < 0},

 extracondition,

 x < (s[p] - eth)/(s'[p] - feed'[p])]
```

Out[•]= $\left\{x < \frac{\text{feed}(p)}{s'(p) - \text{feed}'(p)},\ x < \frac{s(p) - \text{eth}}{s'(p) - \text{feed}'(p)}\right\}$ are equivalent

Inelastic supply has a simple demand-elasticity condition

```
In[•]:= TheoryGuru[{Dt[Equilibrium, x], Equilibrium,
   signconditions, inelasticsupply, x == 0, elasconds0},
  ∂farmersurplus/∂x > ∂subsidyamt/∂x > 0]
```

Out[•]= True

```
In[•]:= TheoryGuru[
  {Dt[Equilibrium, x], Equilibrium, signconditions, inelasticsupply, x == 0},
  SameSign[∂farmersurplus/∂x - ∂subsidyamt/∂x, elasgap] &&
   SameSign[∂farmersurplus/∂x - ∂subsidyamt/∂x, elasweight - 1]]
```

Out[•]= True

```
In[•]:= TheoryGuru[
  {Dt[Equilibrium, x], Equilibrium, signconditions, inelasticsupply, x == 0},
  ∂farmersurplus/∂x == s[p] ∂p/∂x]
```

Out[•]= True

Inelastic ethanol demand

In[]:=
```
Column@signconditions[[2 ;; 3]]
```

Out[]=
$$\text{feed}'[p] \leq 0$$
$$s'[p] \geq 0$$

In[]:=
```
TheoryGuru[{Dt[Equilibrium, x], signconditions[[2 ;; 3]], Not@inelasticsupply,
  eth'[p - x] == 0},
 dp/dx == 0 == dfarmersurplus/dx]
```

Out[]= True

General formula for price impact

In[]:=
```
TheoryGuru[Dt[Equilibrium, x],
 dp/dx == qweight elasweight incidenceparam]
```

Out[]= True

Derivation

In[]:=
```
First@Solve[Dt[Equilibrium, x], Dt[p, x]]
```

Out[]=
$$\left\{\text{Dt}[p,\ x] \to \frac{\text{eth}'[p - x]}{\text{eth}'[p - x] + \text{feed}'[p] - s'[p]}\right\}$$

In[]:=
```
Column@{qweight, elasweight, incidenceparam}
```

Out[]=
$$\frac{\text{eth}[p-x]}{\text{eth}[p-x]+\text{feed}[p]}$$
$$\frac{(\text{eth}[p-x]+\text{feed}[p])\ \text{eth}'[p-x]}{\text{eth}[p-x]\ (\text{eth}'[p-x]+\text{feed}'[p])}$$
$$\frac{\text{eth}'[p-x]+\text{feed}'[p]}{\text{eth}'[p-x]+\text{feed}'[p]-s'[p]}$$

Residual supply curve

In[]:=
```
resids[p_] = s[p] - eth[p - x];
```

In[]:=
```
TheoryOverlap[{}, Equilibrium, feed[p] == resids[p]]
```

Out[]=
$$\{s(p) = \text{eth}(p - x) + \text{feed}(p),\ s(p) - \text{eth}(p - x) = \text{feed}(p)\}\ \text{are equivalent}$$

The ethanol subsidy rate *x* shifts supply in the feed market

```
In[•]:= TheoryGuru[First@signconditions,
         D[resids[p], x] < 0]
Out[•]= True
```

The more elastic is ethanol demand, the more elastic is supply to the feed market

```
In[•]:= resids'[p]
Out[•]= -eth'[p - x] + s'[p]
```

Variable interpretations