Furman ratio without Cobb-Douglas

TheoryGuru applications

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Background

Obama administration economists Furman and Summers claimed that only a fraction of the revenue loss from a corporate tax cut benefits labor. But the standard supply and demand model, which for these purposes is a generalization of long run behavior in the neoclassical growth model, says the opposite.

Here we prove that by machine, without assuming any specific functional form for the aggregate production function. $k$ denotes the aggregate capital stock, $f[k]$ aggregate output gross of depreciation (the aggregate quantity of labor is fixed), and $\tau$ the capital-income tax rate.

Setup

Get@"http://economicreasoning.com"

Proof & Logic Tools 6.1

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Definitions

\[ \text{laborincome}[k_] = f[k] - f'[k] \]
\[ f[k] - k f'[k] \]
lrcapitalequilibrium = (* willingness to pay for capital *)
(1 - τ) (f'[k] - δ) = ρ (* LR willingness to supply it *)
(1 - τ) (-δ + f'[k]) = ρ

signconditions = {δ > 0, ρ > 0, k > 0, f'[k] - δ > 0, 0 ≤ τ < 1, f''[k] ≤ 0};
revenue[τ_, k_] := τ (f'[k] - δ) k
furmanratio[τ_, k_] := d[laborincome[k]]/d[τ] / d[revenue[τ, k]]/d[τ] (* Furman and Summers asserted that it does not make sense for this ratio to exceed one *)

Interesting but not necessary assumptions

elasticcapitaldemand =
D[k f'[k], k] > 0 (* i.e., more capital means more aggregate capital income *)
f'[k] + k f''[k] > 0

wrongsideoflaffercurve = d[revenue[τ, k]]/d[τ] < 0;
fcd[k_] = k^α;

Results

Taxation reduces the capital stock and the amount of labor income

TheoryGuru[{dlrcapitalequilibrium, dρ/dτ ≥ 0 ⇒ dδ/dτ (* the tax does not increase the willingness to supply capital or affect the depreciation rate *),
signconditions[[3 ;; -1]]},
{dk/dτ < 0 ∧ d[laborincome[k]]/d[τ] < 0}
True
Taxation reduces labor income more than it increases revenue

\[
\text{TheoryGuru}\left[\frac{\partial \text{lrcapitalequilibrium}}{\partial \tau}, \text{lrcapitalequilibrium}, \tau > 0, \frac{\partial \rho}{\partial \tau} \geq 0 = \frac{\partial \delta}{\partial \tau}, \text{signconditions}\right], \nonumber
\]

\[
\frac{\partial (\text{laborincome}[k] + \text{revenue}[\tau, k])}{\partial \tau} < 0 \nonumber
\]

True

In the neighborhood of a zero tax rate, this comes from the effect (if any) of the tax on \( \rho \)

\[\text{TheoryGuru}\left[\frac{\partial \text{lrcapitalequilibrium}}{\partial \tau}, \text{lrcapitalequilibrium}, \tau = 0, \frac{\partial \rho}{\partial \tau} \geq 0 = \frac{\partial \delta}{\partial \tau}, \text{signconditions}\right], \nonumber\]

\[\text{SameSign}\left[\frac{\partial (\text{laborincome}[k] + \text{revenue}[\tau, k])}{\partial \tau}, -\frac{\partial \rho}{\partial \tau}\right]\nonumber\]

True

Either the Furman ratio exceeds one or the tax is reducing revenue

\[
\text{TheoryGuru}\left[\frac{\partial \text{lrcapitalequilibrium}}{\partial \tau}, \text{lrcapitalequilibrium}, \tau > 0, \frac{\partial \rho}{\partial \tau} \geq 0 = \frac{\partial \delta}{\partial \tau}, \text{signconditions, elasticcapitaldemand}\right], \nonumber
\]

\[\text{furmanratio}[\tau, k] > 1 \nonumber\]

\[\lor \nonumber\]

\[\text{wrongsideoflaffercurve}\] \nonumber

True

\[
\text{TheoryGuru}\left[\frac{\partial \text{lrcapitalequilibrium}}{\partial \tau}, \text{lrcapitalequilibrium}, \tau > 0, \frac{\partial \rho}{\partial \tau} \geq 0 = \frac{\partial \delta}{\partial \tau}, \text{signconditions}\right], \nonumber
\]

\[\text{furmanratio}[\tau, k] > 1 \nonumber\]

\[\lor \nonumber\]

\[\text{furmanratio}[\tau, k] < 0\] \nonumber

True
TheoryGuru[{{\frac{\text{dlrcapitalequilibrium}}{\text{d}\tau}, \text{lr capitalequilibrium, } \tau = 0, \frac{\text{d}\rho}{\text{d}\tau} \geq 0 = \frac{\text{d}\delta}{\text{d}\tau}},

\text{signconditions, elasticcapitaldemand}},{\text{furmanratio}[\tau, k] \geq 1} \lor \text{wrongsideoflaffercurve}]

True

Cobb-Douglas, with \(0 < \alpha < 1 \land k > 0\), satisfies the sign conditions

\text{FullSimplify}[f'[k] > 0 \land f''[k] \leq 0 \land \text{laborincome}'[k] > 0 \land \text{elasticcapitaldemand} \\
/. f \to \text{fcd} (\ast \text{ replace f with a Cobb-Douglas function } \ast),

\(0 < \alpha < 1 \land k > 0\]

True

Local Cobb-Douglas is a stronger assumption than elasticcapitaldemand

\text{capitalshare}[k_] = \frac{\text{krf'[k]}}{f[k]};

\text{LocalCobbDouglas} = \{0 < \text{capitalshare} < 1, \text{capitalshare}'[k] = 0\};

\text{TheoryOverlap}[\{k > 0, \delta > 0, f[k] > 0, f'[k] \geq \delta, f''[k] \leq 0\}, \text{elasticcapitaldemand, LocalCobbDouglas}]

\(k f''(k) + f'(k) > 0\) is necessary but not sufficient for \(0 < \text{capitalshare} \land \text{capitalshare} < 1 \land \frac{k f''(k)}{f(k)} - \frac{k f'(k)^2}{f(k)^2} + \frac{f'(k)}{f(k)} = 0\)

Variable interpretations

Extensions of this model

Let part of the capital stock escape tax: pdf here and executable Mathematica notebook here.
Let the sellers of final goods have market power: pdf here and executable Mathematica notebook here.
General discussion of extensions here.