

Furman ratio without Cobb-Douglas

TheoryGuru applications

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Background

Obama administration economists Furman and Summers claimed that only a fraction of the revenue loss from a corporate tax cut benefits labor. But the standard supply and demand model, which for these purposes is a generalization of long run behavior in the neoclassical growth model, says the opposite.

Here we prove that by machine, without assuming any specific functional form for the aggregate production function. k denotes the aggregate capital stock, $f[k]$ aggregate output gross of depreciation (the aggregate quantity of labor is fixed), and τ the capital-income tax rate.

Setup

```
Get@"http://economicreasoning.com"
```

Proof & Logic Tools 6.1

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Type ERCommands for a list of commands in the package.

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Definitions

```
laborincome[k_] = f[k] - f'[k] k
```

```
f[k] - k f'[k]
```

```

lrcapitalequilibrium = (* willingness to pay for capital *)
  (1 - τ) (f'[k] - δ) == ρ (* LR willingness to supply it *)
(1 - τ) (-δ + f'[k]) == ρ

signconditions = {δ > 0, ρ > 0, k > 0, f'[k] - δ > 0, 0 ≤ τ < 1, f''[k] ≤ 0};
revenue[τ_, k_] := τ (f'[k] - δ) k
furmanratio[τ_, k_] :=  $\frac{\frac{\partial \text{laborincome}[k]}{\partial \tau}}{\frac{\partial \text{revenue}[\tau, k]}{\partial \tau}}$  (* Furman and Summers
  asserted that it does not make sense for this ratio to exceed one *)

```

Interesting but not necessary assumptions

```

elasticcapitaldemand =
  D[k f'[k], k] > 0 (* i.e., more capital means more aggregate capital income *)
f'[k] + k f''[k] > 0

wrongsideoflaffercurve =  $\frac{\partial \text{revenue}[\tau, k]}{\partial \tau} < 0$ ;
fcd[k_] = kα;

```

Results

Taxation reduces the capital stock and the amount of labor income

```

TheoryGuru[{ $\frac{\partial \text{lrcapitalequilibrium}}{\partial \tau}$ ,
   $\frac{\partial \rho}{\partial \tau} \geq 0 \Rightarrow \frac{\partial \delta}{\partial \tau}$  (* the tax does not increase the willingness
    to supply capital or affect the depreciation rate *)},
  signconditions[[3 ;; -1]]},
 $\frac{\partial k}{\partial \tau} < 0 \wedge \frac{\partial \text{laborincome}[k]}{\partial \tau} < 0$ ]
True

```

Taxation reduces labor income more than it increases revenue

```
TheoryGuru[{ $\frac{d \text{lr capitalequilibrium}}{d \tau}$ , lrcapitalequilibrium,  $\tau > 0$ ,  $\frac{d \rho}{d \tau} \geq 0 \Rightarrow \frac{d \delta}{d \tau}$ ,
  signconditions},
   $\frac{d (\text{laborincome}[k] + \text{revenue}[\tau, k])}{d \tau} < 0$ ]
```

True

In the neighborhood of a zero tax rate, this comes from the effect (if any) of the tax on ρ

```
TheoryGuru[{ $\frac{d \text{lr capitalequilibrium}}{d \tau}$ , lrcapitalequilibrium,  $\tau = 0$ ,  $\frac{d \rho}{d \tau} \geq 0 \Rightarrow \frac{d \delta}{d \tau}$ ,
  signconditions},
  SameSign[ $\frac{d (\text{laborincome}[k] + \text{revenue}[\tau, k])}{d \tau}$ ,  $-\frac{d \rho}{d \tau}$ ]]]
```

True

Either the Furman ratio exceeds one or the tax is reducing revenue

```
TheoryGuru[{ $\frac{d \text{lr capitalequilibrium}}{d \tau}$ , lrcapitalequilibrium,  $\tau > 0$ ,  $\frac{d \rho}{d \tau} \geq 0 \Rightarrow \frac{d \delta}{d \tau}$ ,
  signconditions, elasticcapitaldemand},
  furmanratio[ $\tau, k$ ] > 1
  ∨
  wrongsideoflaffercurve]
```

True

```
TheoryGuru[{ $\frac{d \text{lr capitalequilibrium}}{d \tau}$ , lrcapitalequilibrium,  $\tau > 0$ ,  $\frac{d \rho}{d \tau} \geq 0 \Rightarrow \frac{d \delta}{d \tau}$ ,
  signconditions},
  furmanratio[ $\tau, k$ ] > 1
  ∨
  furmanratio[ $\tau, k$ ] < 0]
```

True

```
TheoryGuru[{ $\frac{d \text{lrcapitalequilibrium}}{d \tau}$ , lrcapitalequilibrium,  $\tau = 0$ ,  $\frac{d \rho}{d \tau} \geq 0 \Rightarrow \frac{d \delta}{d \tau}$ ,
  signconditions, elasticcapitaldemand},
  furmanratio[ $\tau$ , k]  $\geq 1$ 
  v
  wrongsideoflaffercurve]
True
```

Cobb-Douglas, with $0 < \alpha < 1 \wedge k > 0$, satisfies the sign conditions

```
FullSimplify[f'[k] > 0  $\wedge$ 
  f''[k]  $\leq 0 \wedge$ 
  laborincome'[k] > 0  $\wedge$ 
  elasticcapitaldemand
  /. f -> fcd (* replace f with a Cobb-Douglas function *) ,
  0 <  $\alpha < 1 \wedge k > 0$ ]
True
```

Local Cobb-Douglas is a stronger assumption than elasticcapitaldemand

```
capitalshare[k_] =  $\frac{k f'[k]}{f[k]}$ ;
LocalCobbDouglas = {0 < capitalshare < 1, capitalshare'[k] == 0};
TheoryOverlap[{k > 0,  $\delta > 0$ , f[k] > 0, f'[k]  $\geq \delta$ , f''[k]  $\leq 0$ },
  elasticcapitaldemand, LocalCobbDouglas]
k f''(k) + f'(k) > 0 is necessary but not sufficient for
0 < capitalshare  $\wedge$  capitalshare < 1  $\wedge$ 
 $\frac{k f''(k)}{f'(k)} - \frac{k f'(k)^2}{f(k)^2} + \frac{f'(k)}{f(k)} = 0$ 
```

Variable interpretations

Extensions of this model

Let part of the capital stock escape tax: pdf here and executable Mathematica notebook here.
 Let the sellers of final goods have market power: pdf here and executable Mathematica notebook here.
 General discussion of extensions here.