Furman ratio without Cobb-Douglas

For discrete changes in the capital-income tax rate.

TheoryGuru applications

(c) Copyright 2019 by JMJ Economics

Background

Obama administration economists Furman and Summers claimed that only a fraction of the revenue loss from a corporate tax cut benefits labor. But the standard supply and demand model, which for these purposes is a generalization of long run behavior in the neoclassical growth model, says the opposite.

Here we prove that by machine, without assuming any functional form for the aggregate production function. \( k \) denotes the aggregate capital stock, \( f[k] \) aggregate output gross of depreciation (the aggregate quantity of labor is fixed), and \( \tau \) the capital-income tax rate.

Setup

\( \text{In[\_\_\_]} \)  \text{Get}"http://economicreasoning.com"

\textbf{Proof & Logic Tools 6.3}

(c) Copyright 2016, 2017, 2018, 2019 by JMJ Economics

Type **ERCommands** for a list of commands in the package.

**Introduction to Automated Economic Reasoning**

<table>
<thead>
<tr>
<th>Tutorials:</th>
<th>Entering calculus</th>
<th>General Mathematica tips</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Get started</strong></td>
<td>Load extras</td>
<td>Browse examples</td>
</tr>
</tbody>
</table>

Definitions

\( \text{In[\_\_\_]} \)  \text{laborincome[k\_\_]} = f[k] - f'[k] k \)

\( \text{Out[\_\_\_]} \)  \text{f[k] - k f'[k]} \)
$$
\text{lr capitalequilibrium}[\tau, k] := (\text{willingness to pay for capital } \ast) \\
(1 - \tau) f'[k] = \rho + \delta (\ast \text{ LR willingness to supply it } \ast)
$$

$$
\text{signconditions} = \\
\{ \delta > 0, \rho > 0, k_1 > 0, k_2 > 0, 0 \leq \tau_1 < \tau_2 < 1, \text{SameSign}[f'[k_2] - f'[k_1], k_1 - k_2], \\
(* \text{ concave production } *) f'[k_1] (k_1 - k_2) < f[k_1] - f[k_2] < f'[k_2] (k_1 - k_2) \forall k_1 = k_2, \\
\text{SameSign}[\text{labor income}[k_2] - \text{labor income}[k_1], k_2 - k_1], \\
\text{SameSign}[f[k_2] - f[k_1], k_2 - k_1]\};
$$

$$
\text{revenue}[\tau, k] := \tau (f'[k] - \delta) k
$$

$$
\text{furmanratio} := \frac{\text{labor income}[k_2] - \text{labor income}[k_1]}{\text{revenue}[\tau_1, k_1] - \text{revenue}[\tau_2, k_2]}
$$

Interesting but not necessary assumptions

$$
\text{elasticcapitaldemand} = (k_2 f'[k_2] - k_1 f'[k_1]) (k_2 - k_1) \geq 0;
$$

$$
\text{wrongsideoflaffercurve} = (\text{revenue}[\tau_2, k_2] - \text{revenue}[\tau_1, k_1]) (\tau_2 - \tau_1) \leq 0;
$$

Results

Taxation reduces the stock capital and the amount of labor income

$$
\text{TheoryGuru}[[\text{lr capitalequilibrium}[\tau_1, k_1], \text{lr capitalequilibrium}[\tau_2, k_2], \\
\text{Most}@\text{signconditions}], \\
k_2 \prec k_1 \land \text{labor income}[k_1] > \text{labor income}[k_2]]
$$

Out[7] = True

Taxation reduces labor income more than it increases revenue

$$
\text{TheoryGuru}[[\text{lr capitalequilibrium}[\tau_1, k_1], \text{lr capitalequilibrium}[\tau_2, k_2], \\
\text{Most}@\text{signconditions}], \\
\text{revenue}[\tau_1, k_1] + \text{labor income}[k_1] > \text{revenue}[\tau_2, k_2] + \text{labor income}[k_2]]
$$

Out[8] = True
Either the Furman ratio exceeds one or the tax is reducing revenue

\[
\text{furmanratio} > 1 \\
\lor \\
\text{wrongsideoflaffercurve}
\]

\[
\text{furmanratio} < 0
\]