A Local Maximum of a Concave Function
is a Global Maximum

TheoryGuru applications

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Load Economicreasoning package only if it is not already loaded

```
If[Length@Names["PLTools`*"] < 10,
 Get["http://economicreasoning.com"]]
```

Setup

```
fisaconcavefunction = 0 ≤ λ ≤ 1 ⇒ (1 - λ) f[x] + λ f[y] ≤ f[(1 - λ) x + λ y];
(* for any x and y and λ *)

xachieveslocalmax = 0 < λ ≤ ϵ[y] < 1 && f[x] ≥ f[(1 - λ) x + λ y] (*
  x is any allocation.
   y is any allocation not local to x.
   ϵ[y] is the radius bounding what is local to x *);

xachievesglobalmax = f[x] ≥ f[y];
```

Result on Local vs Global Maximum

```
TheoryGuru[{fisaconcavefunction, xachieveslocalmax},
 xachievesglobalmax]
```

```
True
```

Proof is from Beck (2014, Section 8.1), and many others.

Variable interpretations

x and *y* are not necessarily real numbers

x = ;

y = ;

```
TheoryGuru[{fisaconcavefunction, xachieveslocalmax},
 xachievesglobalmax]
```

True

```
Column@MostRecentAssumption
MostRecentHypothesis
```

0 > λ || λ > 1 || λ f[] + (1 - λ) f[] ≤ f[λ  + (1 - λ) ]

f[] ≥ f[λ  + (1 - λ) ]

0 < λ

∈[] < 1

λ ≤ ∈[]

{f[] ≥ f[]}

This model only says that *λ*, f[*x*], *f*[*y*], *f*[*λx*+(1-*λ*)*y*], ϵ[*y*] are real numbers because those five appear as part of inequalities.

x and *y* appear in the model only as arguments of *f* and therefore could be vectors, flags, an abstract object, etc. It is up to the user to ensure that convex combinations of *x* and *y* are meaningful.

TheoryGuru only “knows” that f[*x*], *f*[*y*], and *f*[*λx*+(1-*λ*)*y*] are potentially distinct real numbers because the arguments of *f* are different. e.g., it automatically assumes that *f*[*x*] == *f*[*x*] but not that *f*[*x*] == *f*[*y*].

Setup for Unique Maximum

Strict concavity references equality. If *x* and *y* are, say, vectors rather than scalars, then == should not be used because TheoryGuru interprets the arguments of == as real numbers (e.g., by having the assumption *x* == *y*, then *x* and *y* are automatically interpreted as real numbers).

Here I define my own vector equality function SameVector (you could name it whatever you want), which I interpret as mapping pairs of vector to a Boolean according to whether the two vectors are the same.

```
Remove[x, y]

fisastrictlyconcavefunction = (Not@SameVector[x, y] ∧ 0 < λ < 1) ⇒
   (1 - λ) f[x] + λ f[y] < f[(1 - λ) x + λ y]; (* for any x and y and λ *)

xachievesglobalmax = f[x] ≥ f[(1 - λ) x + λ y];
yachievesglobalmax = f[y] ≥ f[(1 - λ) x + λ y];
```

Result: the Global Maximum of a Strictly Concave Function is Unique

```
TheoryGuru[{0 < λ < 1,
   fisastrictlyconcavefunction, xachievesglobalmax, yachievesglobalmax},

  SameVector[x, y]]
```

```
True
```

```
TheoryGuru[{0 < λ < 1,
   fisastrictlyconcavefunction, xachievesglobalmax},

  SameVector[x, y] ∨ Not@yachievesglobalmax]
```

```
True
```

Proof is from Beck (2014, Section 8.1), and many others.

Use the TheoryBooleans command to see which variable was (variables were) automatically recognized as Booleans.

```
TheoryBooleans
```

```
{SameVector[x, y]}
```

Variable interpretations