Concave and quasiconcave production functions

TheoryGuru applications

Load Economicreasoning package only if it is not already loaded

```
If[Length@Names["PLTools`*"] < 10,
 Get["http://economicreasoning.com"]]
```

```
SetOptions[TheoryGuru, method → "Parallel"];
```

Load other tools by clicking on extras and/or evaluating below

```
If[Not@MemberQ[$ContextPath, "OtherTools`"],
 Get["http://othertools.economicreasoning.com"]]
```

Derivatives at a Point

```
Remove[x, y, z, m]
```

2 inputs

$$\text{fobeyseulersthm} = x\,\frac{\partial f[x, y]}{\partial x} + y\,\frac{\partial f[x, y]}{\partial y} == f[x, y];$$

$$\text{TheoryGuru}\Big[\Big\{\text{QuasiConcaveFunctionQ}[f[x, y], \text{strictly} \to \text{True}],\ x > 0,\ y > 0,\ \text{"}\frac{\partial f[x, y]}{\partial x}\text{"} > 0,\ \text{"}\frac{\partial f[x, y]}{\partial y}\text{"} > 0,\ \text{"}\frac{\partial\ \text{fobeyseulersthm}}{\partial x}\text{"},\ \text{"}\frac{\partial\ \text{fobeyseulersthm}}{\partial y}\text{"}\ (*\ \text{hod}\ 1\ *)\Big\},\ \text{ConcaveFunctionQ@}f[x, y]\Big]$$

```
True
```

Variable interpretations

3 inputs

```
fobeyseulersthm3d = x ∂f[x, y, z]/∂x + y ∂f[x, y, z]/∂y + z ∂f[x, y, z]/∂z == f[x, y, z];

TheoryGuru[{QuasiConcaveFunctionQ[f[x, y, z], strictly → True],
  x > 0, y > 0, z > 0, "∂f[x, y, z]/∂x" > 0, "∂f[x, y, z]/∂y" > 0, "∂f[x, y, z]/∂z" > 0,
  "∂ fobeyseulersthm3d/∂x",
  "∂ fobeyseulersthm3d/∂y", "∂ fobeyseulersthm3d/∂z"(* hod 1 *)},
 ConcaveFunctionQ@f[x, y, z]]
```

```
True
```

Variable interpretations

4 inputs

```
fobeyseulersthm4d = x ∂f[x, y, z, m]/∂x +
   y ∂f[x, y, z, m]/∂y + z ∂f[x, y, z, m]/∂z + m ∂f[x, y, z, m]/∂m == f[x, y, z, m];

TheoryGuru[{QuasiConcaveFunctionQ[f[x, y, z, m], strictly → True],
  x > 0, y > 0, z > 0, m > 0, "∂f[x, y, z, m]/∂x" > 0,
  "∂f[x, y, z, m]/∂y" > 0, "∂f[x, y, z, m]/∂z" > 0, "∂f[x, y, z, m]/∂m" > 0,
  "∂ fobeyseulersthm4d/∂x", "∂ fobeyseulersthm4d/∂y",
  "∂ fobeyseulersthm4d/∂z", "∂ fobeyseulersthm4d/∂m"(* hod 1 *)},
 ConcaveFunctionQ@f[x, y, z, m]]
```

```
True
```

Variable interpretations

Revealed preference derivation (8 points)

a, *b*, *v*, *w* are vectors

Define : $a == \frac{\mu\, v}{f[\mu\, v]}$, $b == \frac{(1-\mu)\, w}{f[(1-\mu)\, w]}$, $\lambda == \frac{f[\mu\, v]}{f[\mu\, v] + f[(1-\mu)\, w]}$

$\lambda\, a == \frac{\mu\, v}{f[\mu\, v] + f[(1-\mu)\, w]}$, $(1-\lambda)\, b == \frac{(1-\mu)\, w}{f[\mu\, v] + f[(1-\mu)\, w]}$

```
definitions = {λ == f[μ v] / (f[μ v] + f[(1 - μ) w])};

positivef = {f[μ v] > 0, f[(1 - μ) w] > 0};

homogeneousf =
  {f[a] == f[b] == 1, f[μ v + (1 - μ) w] == (f[μ v] + f[(1 - μ) w]) f[λ a + (1 - λ) b],
   μ > 0 ⇒ f[μ v] == μ f[v], μ < 1 ⇒ f[(1 - μ) w] == (1 - μ) f[w]};

quasiconcavef = {(f[a] == f[b] && 0 < λ < 1) ⇒ f[λ a + (1 - λ) b] ≥ 1};

concavef = {0 < μ < 1 ⇒ f[μ v + (1 - μ) w] ≥ μ f[v] + (1 - μ) f[w]};

TheoryGuru[{definitions, positivef, homogeneousf, quasiconcavef}, concavef]

True
```

Variable interpretations