

Laffer curve example

TheoryGuru applications

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Load Economicreasoning package only if it is not already loaded

```
If[Length@Names["PLTools`*"] < 10,  
  Get["http://economicreasoning.com"]]
```

Setup

```
basicassumptions = {0 <  $\tau_L$  <  $\tau_H$  < 1,  $w_L$  > 0,  $w_H$  > 0,  $n_L$  > 0,  $n_H$  > 0,  $c_L$  > 0,  $c_H$  > 0,  
   $\tau_L n_L w_L == \tau_H n_H w_H$ ,  
   $(c_L - c_H) (n_L - n_H) > 0$ ,  $(w_L - w_H) (n_L - n_H) \leq 0$ ,  $(n_L - n_H) w_L \leq c_L - c_H \leq (n_L - n_H) w_H$ ,  
   $u_L > u_{Lb}$ ,  $(u_{Lb} - u_H) (c_{Lb} - c_H) > 0$ ,  $c_L - c_{Lb} == (n_L - n_H) (1 - \tau_L) w_L$ ,  
   $u_H > u_{Hb}$ ,  $(u_{Hb} - u_L) (c_{Hb} - c_L) > 0$ ,  $c_H - c_{Hb} == (n_H - n_L) (1 - \tau_H) w_H$ };  
  
mrsassumptions = { $m_L == (1 - \tau_L) w_L$ ,  $m_H == (1 - \tau_H) w_H$ };  
  
ldemandiselastic =  $(w_L n_L - w_H n_H) (n_L - n_H) \geq 0$ ;  
  
bothnormalgoods =  $(m_L - m_H) (n_L - n_H) > 0$ ;
```

basicassumptions are the assumptions/assertions necessary for the queries posed in this notebook. Other queries, e.g., with $\tau_L == \tau_H$ may require additional assumptions. See the Laffer-curve section of MultipleorUniqueEquilibrium.nb (pdf here) for a more extensive assumption specification for this model.

Analytical Results

The high-tax equilibrium could have more work and utility

```
TheoryGuru[basicassumptions,  $n_L > n_H$ ]
```

```
TheoryGuru[basicassumptions,  $u_L > u_H$ ]
```

Restrictions that guarantee more utility at low tax

`TheoryGuru[{basicassumptions, mrsassumptions, bothnormalgoods}, nL > nH && uL > uH]`

`TheoryGuru[{basicassumptions, ldemandiselastic}, nL > nH && uL > uH]`

Variable interpretations

Numerical Example

Preferences

$$u[c_, n_] := \frac{3}{4} \text{Log}[c] - \frac{n}{50} - \frac{2}{3} (-1 + 2c)^2 n^3 + 1$$

$$\text{mrs}[c_, n_] := - \frac{u^{(0,1)}[c, n]}{u^{(1,0)}[c, n]}$$

Technology (kinked production function)

`a0 = 1 / 5; a2 = 4 / 5;`

`a1 = a /. First@NSolve[mrs[a + a0, 1] == a, a]`

`0.44206`

`f[n_] := Min[a1 n + a0, a2 n]`

$$\text{utilcut} = u\left[f\left[\frac{a0}{a2 - a1}\right], \frac{a0}{a2 - a1}\right]$$

`0.383625`

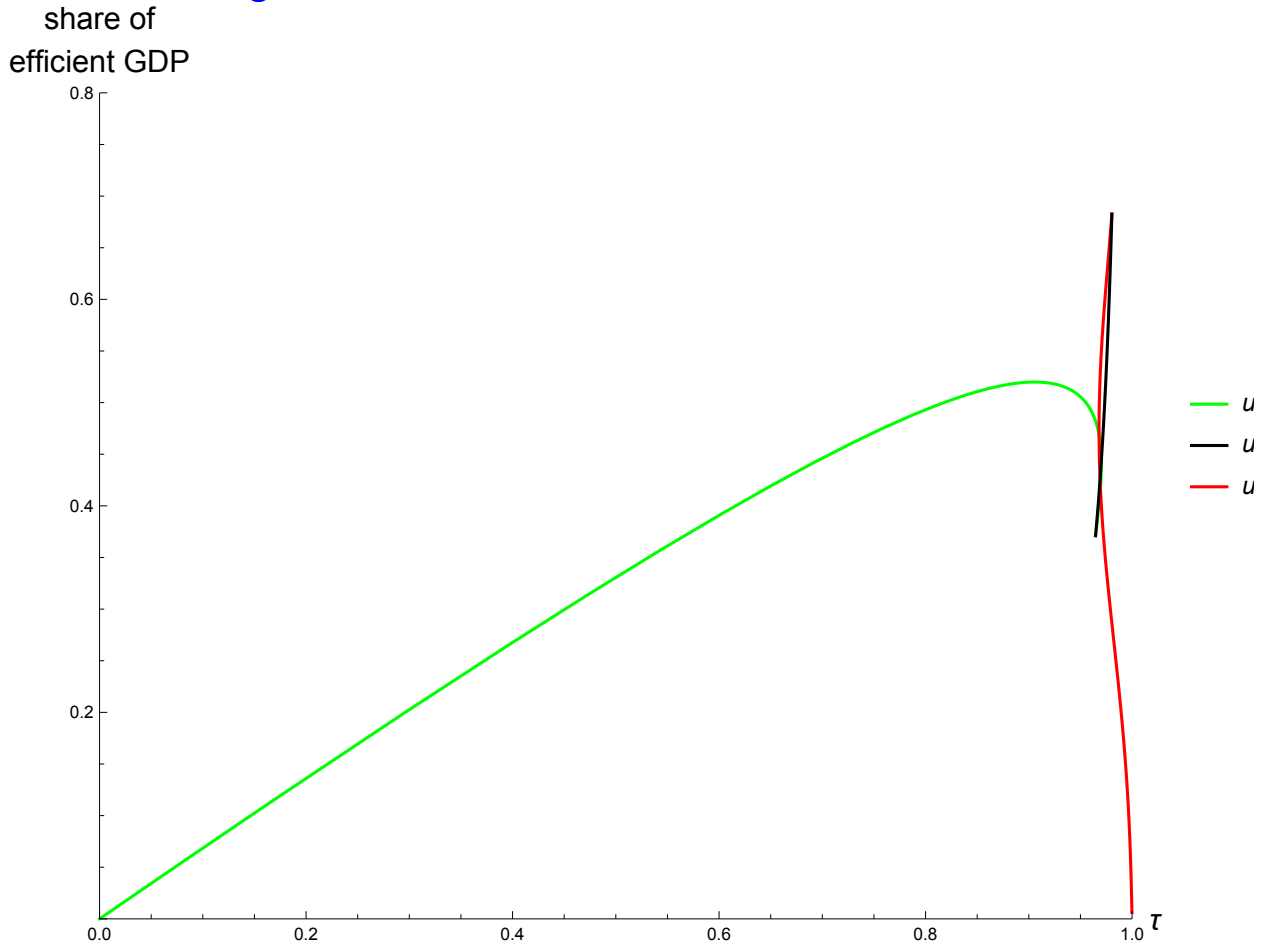
Plot entire Laffer curve for the numerical example

```

ParametricPlot[ $\left\{1 - \frac{\text{mrs}[f[n], n]}{f'[n]}, \frac{f'[n] - \text{mrs}[f[n], n]}{f[1]} n\right\}$ , {n,  $\frac{a_0}{a_2 - a_1}$ , 1},
  PlotLabel → Style["Figure 4a. The Laffer curve for transfers", {Large, Blue}],
  AxesLabel → (Style[#, FontSize → 16] & /@ {"τ", "share of inefficient GDP"}),
  PlotRange → {{0, 1}, {0, 4/5}}, ImageSize → 600, PlotStyle → Green] ~ Show ~
ParametricPlot[ $\left\{1 - \frac{\text{mrs}[f[n], n]}{f'[n]}, \frac{f'[n] - \text{mrs}[f[n], n]}{f[1]} n\right\}$ , {n, 1/200,  $\frac{a_0}{a_2 - a_1}$ },
  PlotRange → {{0, 1}, {0, 4/5}}, ImageSize → 600, PlotStyle → Red] ~
Show~Plot[{2, 3, 2}, {tau, 19/20, 1}, PlotStyle → {Green, Black, Red},
  PlotLegends → {"u(c, n) > u_kink", "u(c, n) = u_kink", "u(c, n) ≤ u_kink"}] ~ Show ~
ParametricPlot[ $\left\{1 - \frac{\text{mrs}\left[f\left[\frac{a_0}{a_2 - a_1}\right], \frac{a_0}{a_2 - a_1}\right]}{w}, \frac{w - \text{mrs}\left[f\left[\frac{a_0}{a_2 - a_1}\right], \frac{a_0}{a_2 - a_1}\right]}{f[1]} \frac{a_0}{a_2 - a_1}\right\}$ ,
  {w, a1, a2}, PlotRange → {{0, 1}, {0, 4/5}}, ImageSize → 600, PlotStyle → Black]

```

Figure 4a. The Laffer curve for transfers

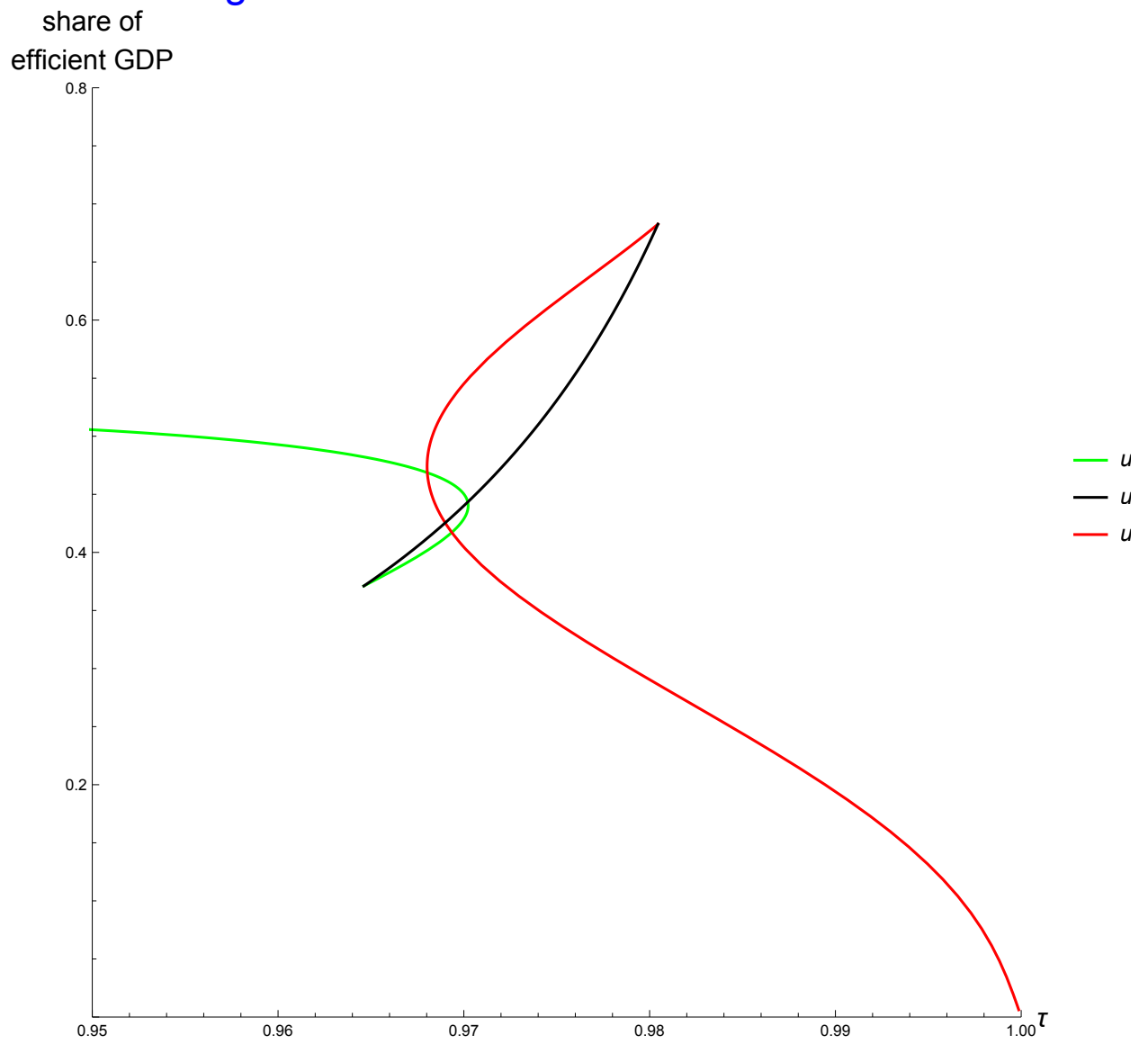


```

ParametricPlot[ $\left\{1 - \frac{\text{mrs}[f[n], n]}{f'[n]}, \frac{f'[n] - \text{mrs}[f[n], n]}{f[1]} n\right\}$ , {n,  $\frac{a_0}{a_2 - a_1}$ , 1},
  PlotLabel → Style["Figure 4b. The Laffer curve for transfers", {Large, Blue}],
  AxesLabel → (Style[#, FontSize → 16] & /@ {"τ", "share of\nefficient GDP"}),
  PlotRange → {{19/20, 1}, {0, 4/5}}, ImageSize → 600,
  PlotStyle → Green, AspectRatio → 1] ~ Show ~
ParametricPlot[ $\left\{1 - \frac{\text{mrs}[f[n], n]}{f'[n]}, \frac{f'[n] - \text{mrs}[f[n], n]}{f[1]} n\right\}$ , {n, 1/200,  $\frac{a_0}{a_2 - a_1}$ },
  PlotLabel → Style["The Laffer curve for transfers", {Large, Blue}],
  AxesLabel → (Style[#, FontSize → 16] & /@ {"τ", "share of\nefficient GDP"}),
  PlotRange → {{19/20, 1}, {0, 4/5}}, ImageSize → 600,
  PlotStyle → Red, AspectRatio → 1] ~ Show ~
Plot[{2, 3, 2}, {tau, 19/20, 1}, PlotStyle → {Green, Black, Red},
  PlotLegends → {"u(c, n) > ukink", "u(c, n) = ukink", "u(c, n) ≤ ukink"}] ~ Show ~
ParametricPlot[ $\left\{1 - \frac{\text{mrs}\left[f\left[\frac{-a_0}{a_2 - a_1}\right], \frac{-a_0}{a_2 - a_1}\right]}{w}, \frac{w - \text{mrs}\left[f\left[\frac{-a_0}{a_2 - a_1}\right], \frac{-a_0}{a_2 - a_1}\right]}{f[1]} \frac{a_0}{a_2 - a_1}\right\}$ ,
  {w, a1, a2}, PlotRange → {{0, 1}, {0, 4/5}}, ImageSize → 600, PlotStyle → Black]

```

Figure 4b. The Laffer curve for transfers



Take a pair equilibria to confirm that
 (a) both goods are not normal,
 (b) labor demand is not elastic,
 (c) $\tau_L < \tau_H$, and
 (d) utility is higher at the high tax rate
 in that range

```

examplepaairofpoints = {mL → mrs[cL, nL], mH → mrs[cH, nH], uL → u[cL, nL], uH → u[cH, nH],
  cL → f[nL], cH → f[nH], wL → f'[nL], wH → f'[nH], τL → 1 -  $\frac{m_L}{w_L}$ , τH → 1 -  $\frac{m_H}{w_H}$ ,
  nL →  $\frac{2}{3} \frac{a_0}{a_2 - a_1}$  (* below production-function kink *),
  nH →  $\frac{6}{5} \frac{a_0}{a_2 - a_1}$  (* above production-function kink *)};

{Not@bothnormalgoods, Not@ldemandiselastic, τL < τH, SameSign[uH - uL, τH - τL]} //.
examplepaairofpoints

{True, True, True, True}

```