

Method of Moments

TheoryGuru applications

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Load Economicreasoning package only if it is not already loaded

```
If[Length@Names["PLTools`*"] < 10,  
  Get["http://economicreasoning.com"]]
```

Notes

In the Wolfram Language, $x.y$ refers to the tensor DOT PRODUCT, NOT scalar multiplication. For TheoryGuru purposes, tensor means vector, so that the result of $x.y$ is a scalar.

Any equation or inequality used as an argument to TheoryGuru must refer to scalars. Those inequalities (or equations) can contain vector dot products, in which case vector mode is automatically turned on.

Setup

```
numregressors = 4;  
regcoefs = Table[ $\beta[i]$ , {i, numregressors}]  
{ $\beta[1]$ ,  $\beta[2]$ ,  $\beta[3]$ ,  $\beta[4]$ }  
indvars = Table[x[i], {i, numregressors}]  
{x[1], x[2], x[3], x[4]}  
regmodel = indvars.regcoefs + e  
 $\epsilon + x[1] \beta[1] + x[2] \beta[2] + x[3] \beta[3] + x[4] \beta[4]$   
momentconditions = Assuming[Alternatives@@regcoefs  $\in$  Reals,  
  Thread[Dot[#, y == regmodel], Equal] & /@ indvars // TensorExpand];
```

Column@momentconditions

```

x[1].y - x[1].ε - x[1].x[1] β[1] - x[1].x[2] β[2] - x[1].x[3] β[3] - x[1].x[4] β[4] == 0
x[2].y - x[2].ε - x[2].x[1] β[1] - x[2].x[2] β[2] - x[2].x[3] β[3] - x[2].x[4] β[4] == 0
x[3].y - x[3].ε - x[3].x[1] β[1] - x[3].x[2] β[2] - x[3].x[3] β[3] - x[3].x[4] β[4] == 0
x[4].y - x[4].ε - x[4].x[1] β[1] - x[4].x[2] β[2] - x[4].x[3] β[3] - x[4].x[4] β[4] == 0

```

```
coefsoln = FullSimplify@Values@First@Solve[momentconditions, regcoefs];
```

```
olsestimate =
```

```
FullSimplify@Values@First@Solve[momentconditions /. {Dot[_ , ε] → 0}, regcoefs];
```

```
Assuming[Alternatives@@regcoefs ∈ Reals,
```

```
momentconditions /. {y → regmodel} // TensorExpand]
```

```
{True, True, True, True}
```

```
tmp1 = Values@First@Solve[momentconditions, regcoefs] /. y → regmodel;
```

```
FullSimplify@Assuming[Alternatives@@regcoefs ∈ Reals, TensorExpand@First@tmp1]
```

```
β[1]
```

First@tmp1

$$\begin{aligned}
& - \left(\left(- \left(-x[1] \cdot \epsilon + x[1] \cdot (\epsilon + x[1] \beta[1] + x[2] \beta[2] + x[3] \beta[3] + x[4] \beta[4]) \right) \right) x[2] \cdot x[4] + \right. \\
& \quad \left. x[1] \cdot x[4] \left(x[2] \cdot \epsilon - x[2] \cdot (\epsilon + x[1] \beta[1] + x[2] \beta[2] + x[3] \beta[3] + \right. \right. \\
& \quad \quad \left. \left. x[4] \beta[4]) \right) \right) \left(x[1] \cdot x[4] x[3] \cdot x[3] - x[1] \cdot x[3] x[3] \cdot x[4] \right) + \\
& \quad \left(x[1] \cdot x[4] x[2] \cdot x[3] - x[1] \cdot x[3] x[2] \cdot x[4] \right) \left(\left(-x[1] \cdot \epsilon + x[1] \cdot (\epsilon + \right. \right. \\
& \quad \quad \left. \left. x[1] \beta[1] + x[2] \beta[2] + x[3] \beta[3] + x[4] \beta[4]) \right) x[3] \cdot x[4] + x[1] \cdot x[4] \right. \\
& \quad \quad \left. \left(x[3] \cdot \epsilon - x[3] \cdot (\epsilon + x[1] \beta[1] + x[2] \beta[2] + x[3] \beta[3] + x[4] \beta[4]) \right) \right) \\
& \quad \left(x[1] \cdot x[4] \left(x[2] \cdot x[3] x[4] \cdot x[2] - x[2] \cdot x[2] x[4] \cdot x[3] \right) + \right. \\
& \quad \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] x[4] \cdot x[2] + x[2] \cdot x[2] x[4] \cdot x[4] \right) + \\
& \quad \quad \left. x[1] \cdot x[2] \left(x[2] \cdot x[4] x[4] \cdot x[3] - x[2] \cdot x[3] x[4] \cdot x[4] \right) \right) + \\
& \quad \left(x[1] \cdot x[4] \left(x[2] \cdot x[3] x[3] \cdot x[2] - x[2] \cdot x[2] x[3] \cdot x[3] \right) + \right. \\
& \quad \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] x[3] \cdot x[2] + x[2] \cdot x[2] x[3] \cdot x[4] \right) + \\
& \quad \quad \left. x[1] \cdot x[2] \left(x[2] \cdot x[4] x[3] \cdot x[3] - x[2] \cdot x[3] x[3] \cdot x[4] \right) \right) \\
& \quad \left(- \left(-x[1] \cdot \epsilon + x[1] \cdot (\epsilon + x[1] \beta[1] + x[2] \beta[2] + x[3] \beta[3] + x[4] \beta[4]) \right) x[2] \cdot x[4] + \right. \\
& \quad \quad \left. x[1] \cdot x[4] \left(x[2] \cdot \epsilon - x[2] \cdot (\epsilon + x[1] \beta[1] + x[2] \beta[2] + x[3] \beta[3] + x[4] \beta[4]) \right) \right) \\
& \quad \left(x[1] \cdot x[4] x[4] \cdot x[3] - x[1] \cdot x[3] x[4] \cdot x[4] \right) + \\
& \quad \left(x[1] \cdot x[4] x[2] \cdot x[3] - x[1] \cdot x[3] x[2] \cdot x[4] \right) \\
& \quad \left(\left(-x[1] \cdot \epsilon + x[1] \cdot (\epsilon + x[1] \beta[1] + x[2] \beta[2] + x[3] \beta[3] + x[4] \beta[4]) \right) \right) \\
& \quad \quad x[4] \cdot x[4] + x[1] \cdot x[4] \\
& \quad \quad \left. \left(x[4] \cdot \epsilon - x[4] \cdot (\epsilon + x[1] \beta[1] + x[2] \beta[2] + x[3] \beta[3] + x[4] \beta[4]) \right) \right) \right) / \\
& \quad \left(x[1] \cdot x[4] \left(\left(x[1] \cdot x[4] \left(x[2] \cdot x[3] x[3] \cdot x[2] - x[2] \cdot x[2] x[3] \cdot x[3] \right) + \right. \right. \right. \\
& \quad \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] x[3] \cdot x[2] + x[2] \cdot x[2] x[3] \cdot x[4] \right) + \\
& \quad \quad \left. x[1] \cdot x[2] \left(x[2] \cdot x[4] x[3] \cdot x[3] - x[2] \cdot x[3] x[3] \cdot x[4] \right) \right) \\
& \quad \left(x[1] \cdot x[4] \left(x[2] \cdot x[3] x[4] \cdot x[1] - x[2] \cdot x[1] x[4] \cdot x[3] \right) + \right. \\
& \quad \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] x[4] \cdot x[1] + x[2] \cdot x[1] x[4] \cdot x[4] \right) + \\
& \quad \quad \left. x[1] \cdot x[1] \left(x[2] \cdot x[4] x[4] \cdot x[3] - x[2] \cdot x[3] x[4] \cdot x[4] \right) \right) - \\
& \quad \left(x[1] \cdot x[4] \left(x[2] \cdot x[3] x[3] \cdot x[1] - x[2] \cdot x[1] x[3] \cdot x[3] \right) + \right. \\
& \quad \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] x[3] \cdot x[1] + x[2] \cdot x[1] x[3] \cdot x[4] \right) + \\
& \quad \quad \left. x[1] \cdot x[1] \left(x[2] \cdot x[4] x[3] \cdot x[3] - x[2] \cdot x[3] x[3] \cdot x[4] \right) \right) \\
& \quad \left(x[1] \cdot x[4] \left(x[2] \cdot x[3] x[4] \cdot x[2] - x[2] \cdot x[2] x[4] \cdot x[3] \right) + \right. \\
& \quad \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] x[4] \cdot x[2] + x[2] \cdot x[2] x[4] \cdot x[4] \right) + \\
& \quad \quad \left. x[1] \cdot x[2] \left(x[2] \cdot x[4] x[4] \cdot x[3] - x[2] \cdot x[3] x[4] \cdot x[4] \right) \right) \right)
\end{aligned}$$

First@coefsoln

First@olsestimate

$$\begin{aligned}
& - \left(\left(x[1] \cdot x[4] \left(x[2] \cdot x[3] x[3] \cdot x[2] - x[2] \cdot x[2] x[3] \cdot x[3] \right) + \right. \right. \\
& \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] x[3] \cdot x[2] + x[2] \cdot x[2] x[3] \cdot x[4] \right) + \\
& \quad \left. \left. x[1] \cdot x[2] \left(x[2] \cdot x[4] x[3] \cdot x[3] - x[2] \cdot x[3] x[3] \cdot x[4] \right) \right) \right) \\
& \left(x[1] \cdot x[4] x[2] \cdot x[3] - x[1] \cdot x[3] x[2] \cdot x[4] \right) \\
& \quad \left(x[1] \cdot x[4] \left(-x[4] \cdot y + x[4] \cdot \epsilon \right) + \left(x[1] \cdot y - x[1] \cdot \epsilon \right) x[4] \cdot x[4] \right) - \\
& \quad \left(x[1] \cdot x[4] \left(-x[2] \cdot y + x[2] \cdot \epsilon \right) + \left(x[1] \cdot y - x[1] \cdot \epsilon \right) x[2] \cdot x[4] \right) \\
& \quad \left(x[1] \cdot x[4] x[4] \cdot x[3] - x[1] \cdot x[3] x[4] \cdot x[4] \right) \Big) - \\
& \left(x[1] \cdot x[4] x[2] \cdot x[3] - x[1] \cdot x[3] x[2] \cdot x[4] \right) \\
& \quad \left(x[1] \cdot x[4] \left(-x[3] \cdot y + x[3] \cdot \epsilon \right) + \left(x[1] \cdot y - x[1] \cdot \epsilon \right) x[3] \cdot x[4] \right) - \\
& \quad \left(x[1] \cdot x[4] \left(-x[2] \cdot y + x[2] \cdot \epsilon \right) + \left(x[1] \cdot y - x[1] \cdot \epsilon \right) x[2] \cdot x[4] \right) \\
& \quad \left(x[1] \cdot x[4] x[3] \cdot x[3] - x[1] \cdot x[3] x[3] \cdot x[4] \right) \Big) \\
& \left(x[1] \cdot x[4] \left(x[2] \cdot x[3] x[4] \cdot x[2] - x[2] \cdot x[2] x[4] \cdot x[3] \right) + \right. \\
& \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] x[4] \cdot x[2] + x[2] \cdot x[2] x[4] \cdot x[4] \right) + \\
& \quad \left. x[1] \cdot x[2] \left(x[2] \cdot x[4] x[4] \cdot x[3] - x[2] \cdot x[3] x[4] \cdot x[4] \right) \right) \Big) / \\
& \left(x[1] \cdot x[4] \left(x[1] \cdot x[4] \left(x[2] \cdot x[3] x[3] \cdot x[2] - x[2] \cdot x[2] x[3] \cdot x[3] \right) + \right. \right. \\
& \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] x[3] \cdot x[2] + x[2] \cdot x[2] x[3] \cdot x[4] \right) + \\
& \quad \left. x[1] \cdot x[2] \left(x[2] \cdot x[4] x[3] \cdot x[3] - x[2] \cdot x[3] x[3] \cdot x[4] \right) \right) \\
& \quad \left(x[1] \cdot x[4] \left(x[2] \cdot x[3] x[4] \cdot x[1] - x[2] \cdot x[1] x[4] \cdot x[3] \right) + \right. \\
& \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] x[4] \cdot x[1] + x[2] \cdot x[1] x[4] \cdot x[4] \right) + \\
& \quad \left. x[1] \cdot x[1] \left(x[2] \cdot x[4] x[4] \cdot x[3] - x[2] \cdot x[3] x[4] \cdot x[4] \right) \right) - \\
& \quad \left(x[1] \cdot x[4] \left(x[2] \cdot x[3] x[3] \cdot x[1] - x[2] \cdot x[1] x[3] \cdot x[3] \right) + \right. \\
& \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] x[3] \cdot x[1] + x[2] \cdot x[1] x[3] \cdot x[4] \right) + \\
& \quad \left. x[1] \cdot x[1] \left(x[2] \cdot x[4] x[3] \cdot x[3] - x[2] \cdot x[3] x[3] \cdot x[4] \right) \right) \\
& \quad \left(x[1] \cdot x[4] \left(x[2] \cdot x[3] x[4] \cdot x[2] - x[2] \cdot x[2] x[4] \cdot x[3] \right) + \right. \\
& \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] x[4] \cdot x[2] + x[2] \cdot x[2] x[4] \cdot x[4] \right) + \\
& \quad \left. x[1] \cdot x[2] \left(x[2] \cdot x[4] x[4] \cdot x[3] - x[2] \cdot x[3] x[4] \cdot x[4] \right) \right) \Big) \Big)
\end{aligned}$$

$$\begin{aligned}
& - \left(\left(x[1] \cdot x[4] \left(x[2] \cdot x[3] \cdot x[3] \cdot x[2] - x[2] \cdot x[2] \cdot x[3] \cdot x[3] \right) + \right. \right. \\
& \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] \cdot x[3] \cdot x[2] + x[2] \cdot x[2] \cdot x[3] \cdot x[4] \right) + \\
& \quad \left. \left. x[1] \cdot x[2] \left(x[2] \cdot x[4] \cdot x[3] \cdot x[3] - x[2] \cdot x[3] \cdot x[3] \cdot x[4] \right) \right) \right) \\
& \quad \left(x[2] \cdot x[4] \left(x[1] \cdot x[3] \cdot x[4] \cdot y - x[1] \cdot y \cdot x[4] \cdot x[3] \right) + \right. \\
& \quad x[1] \cdot x[4] \left(-x[2] \cdot x[3] \cdot x[4] \cdot y + x[2] \cdot y \cdot x[4] \cdot x[3] \right) + \\
& \quad \left. \left(-x[1] \cdot x[3] \cdot x[2] \cdot y + x[1] \cdot y \cdot x[2] \cdot x[3] \right) \cdot x[4] \cdot x[4] \right) - \\
& \quad \left(x[2] \cdot x[4] \left(x[1] \cdot x[3] \cdot x[3] \cdot y - x[1] \cdot y \cdot x[3] \cdot x[3] \right) + \right. \\
& \quad x[1] \cdot x[4] \left(-x[2] \cdot x[3] \cdot x[3] \cdot y + x[2] \cdot y \cdot x[3] \cdot x[3] \right) + \\
& \quad \left. \left(-x[1] \cdot x[3] \cdot x[2] \cdot y + x[1] \cdot y \cdot x[2] \cdot x[3] \right) \cdot x[3] \cdot x[4] \right) \\
& \quad \left(x[1] \cdot x[4] \left(x[2] \cdot x[3] \cdot x[4] \cdot x[2] - x[2] \cdot x[2] \cdot x[4] \cdot x[3] \right) + \right. \\
& \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] \cdot x[4] \cdot x[2] + x[2] \cdot x[2] \cdot x[4] \cdot x[4] \right) + \\
& \quad \left. \left. x[1] \cdot x[2] \left(x[2] \cdot x[4] \cdot x[4] \cdot x[3] - x[2] \cdot x[3] \cdot x[4] \cdot x[4] \right) \right) \right) / \\
& \quad \left(x[1] \cdot x[4] \left(x[2] \cdot x[3] \cdot x[3] \cdot x[2] - x[2] \cdot x[2] \cdot x[3] \cdot x[3] \right) + \right. \\
& \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] \cdot x[3] \cdot x[2] + x[2] \cdot x[2] \cdot x[3] \cdot x[4] \right) + \\
& \quad x[1] \cdot x[2] \left(x[2] \cdot x[4] \cdot x[3] \cdot x[3] - x[2] \cdot x[3] \cdot x[3] \cdot x[4] \right) \right) \\
& \quad \left(x[1] \cdot x[4] \left(x[2] \cdot x[3] \cdot x[4] \cdot x[1] - x[2] \cdot x[1] \cdot x[4] \cdot x[3] \right) + \right. \\
& \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] \cdot x[4] \cdot x[1] + x[2] \cdot x[1] \cdot x[4] \cdot x[4] \right) + \\
& \quad \left. \left. x[1] \cdot x[1] \left(x[2] \cdot x[4] \cdot x[4] \cdot x[3] - x[2] \cdot x[3] \cdot x[4] \cdot x[4] \right) \right) - \right. \\
& \quad \left(x[1] \cdot x[4] \left(x[2] \cdot x[3] \cdot x[3] \cdot x[1] - x[2] \cdot x[1] \cdot x[3] \cdot x[3] \right) + \right. \\
& \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] \cdot x[3] \cdot x[1] + x[2] \cdot x[1] \cdot x[3] \cdot x[4] \right) + \\
& \quad \left. \left. x[1] \cdot x[1] \left(x[2] \cdot x[4] \cdot x[3] \cdot x[3] - x[2] \cdot x[3] \cdot x[3] \cdot x[4] \right) \right) \right) \\
& \quad \left(x[1] \cdot x[4] \left(x[2] \cdot x[3] \cdot x[4] \cdot x[2] - x[2] \cdot x[2] \cdot x[4] \cdot x[3] \right) + \right. \\
& \quad x[1] \cdot x[3] \left(-x[2] \cdot x[4] \cdot x[4] \cdot x[2] + x[2] \cdot x[2] \cdot x[4] \cdot x[4] \right) + \\
& \quad \left. \left. x[1] \cdot x[2] \left(x[2] \cdot x[4] \cdot x[4] \cdot x[3] - x[2] \cdot x[3] \cdot x[4] \cdot x[4] \right) \right) \right)
\end{aligned}$$