Preference changes: risk aversion example

TheoryGuru applications

Load Economicreasoning package only if it is not already loaded

```
If[Length@Names["PLTools`*"] < 10,
 Get["http://economicreasoning.com"]]
```

Notes

ϵ and the partial derivatives of u are each automatically recognized as vectors with length equal to the number of possible investment outcomes. That number must not be less than one but is otherwise arbitrary.

In the Wolfram Language, x.y refers to the tensor DOT PRODUCT, NOT scalar multiplication. For TheoryGuru purposes, tensor means vector, so that the result of x.y is a scalar.

Setup

Take wealth w and allocate x of it to a risky activity that has gross return ϵ.
The utility function is a function of total income and a preference parameter r.

$$\text{optimum} = \frac{\partial\,(p.u(w + (\epsilon - 1)\;x,\;r))}{\partial x} == 0;$$

Vary the preference parameter r holding w, ϵ and p constant.

```
ComparativeStatic = Dt[optimum, r] /. (Dt[w, r] | Dt[ϵ, r] | Dt[p, r]._) → 0;
```

$$\texttt{SecondOrderCondition} = \frac{\partial^2 (p.u(w+(\epsilon-1)\,x,\,r))}{\partial x^2} < 0;$$

$$\texttt{DefineAverages} = \left\{\texttt{avmu} == p.u^{(1,0)}[w+x\,(-1+\epsilon),\,r] > 0,\ \texttt{aveps} == p.\epsilon > 1\right\};$$

$$\texttt{NegativeCorrelationBetweenReturnandMU} = (\epsilon-\texttt{aveps}).\left(u^{(1,0)}[w+x\,(-1+\epsilon),\,r]-\texttt{avmu}\right) < 0;$$

$$\texttt{PrefChangeThatWidensMUGaps} = D\left[(\epsilon-1).u^{(1,0)}[w+x\,(-1+\epsilon),\,r],\,r\right] < 0;$$

Results

At the optimum, there is a negative correlation (across investment outcomes) between the return and marginal utility

```
TheoryGuru[{optimum, ProbabilityVector@p, DefineAverages},
 NegativeCorrelationBetweenReturnandMU]
```

```
True
```

A preference change that widens these marginal utility gaps -- such as an increase in risk aversion -- reduces the optimal amount *x* to allocate to the risky activity

$$\texttt{TheoryGuru}\Big[\Big\{\texttt{ComparativeStatic},\ \texttt{PrefChangeThatWidensMUGaps},\ \texttt{SecondOrderCondition},\ \texttt{ProbabilityVector@p},\ u^{(1,0)}[w+x\,(-1+\epsilon),\,r].u^{(1,0)}[w+x\,(-1+\epsilon),\,r] > 0\Big\},\ \frac{dx}{dr} < 0\Big]$$

Your model has 7 vectors (not including probability vectors):

$$\left\{\epsilon,\ \frac{\partial u(w+x\,(\epsilon-1),\,r)}{\partial w+x\,(\epsilon-1)},\ \frac{\partial^2 u(w+x\,(\epsilon-1),\,r)}{\partial w+x\,(\epsilon-1)\,\partial r},\ \frac{\partial^2 u(w+x\,(\epsilon-1),\,r)}{\partial (w+x\,(\epsilon-1))^2}\right\}$$

plus the vectors $\left\{\epsilon\,\frac{\partial^2 u(w+x\,(\epsilon-1),\,r)}{\partial w+x\,(\epsilon-1)\,\partial r},\ \epsilon\,\frac{\partial^2 u(w+x\,(\epsilon-1),\,r)}{\partial (w+x\,(\epsilon-1))^2},\ \epsilon^2\,\frac{\partial^2 u(w+x\,(\epsilon-1),\,r)}{\partial (w+x\,(\epsilon-1))^2}\right\}$

formed from element-by-element multiplication.

```
True
```

Variable interpretations