The Price of Skill Changes the Selection Rule, Demonstrated with the Roy Model

TheoryGuru applications

Background

Below we see that women and men appeared to become more equal in terms of wages, while at the same time wages become less equal among men. Is this a puzzle?

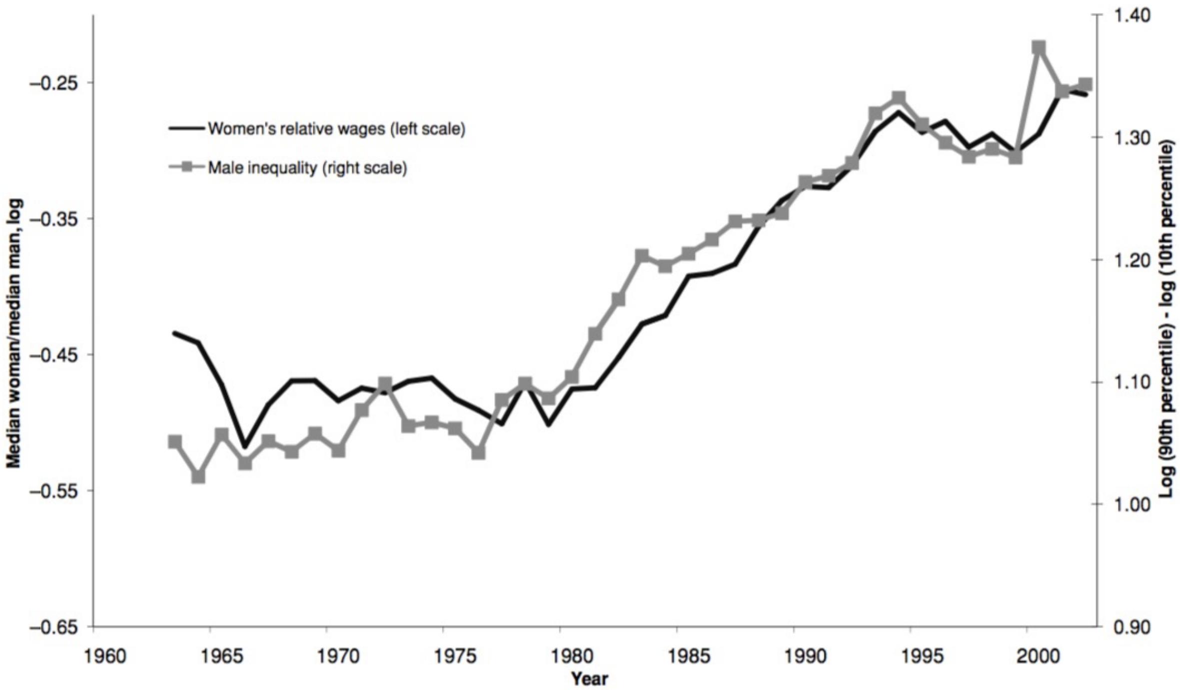

FIGURE I
Wage Inequality within and between Genders

Source: Mulligan and Rubinstein (2008)

Women choose between market and nonmarket activities on the basis of what they would earn in each activity, whose natural logarithms are proportional to their skills in those sectors, *h* and *r*, respectively. The joint distribution of those skills has density $f[h, r]$. We represent the log market wage as $w == \sigma h + \mu_w$ and the log nonmarket wage as $R == r + \mu_R$. We do not assume any functional form for f, or equate the skills with measurable quantities such as IQ, so any monotone transformation of h or r could be used instead.

In order to analyze the historical trends shown above, we consider a comparative static that increases the variance of log market wages (a.k.a., more "inequality"), namely an increase in the parameter σ. Market labor-supply shifts are modeled as comparative statics with respect to μ_R.

The impact of inequality on the average log wage among working women can be decomposed into:
(i) the impact of inequality at a given average skill for working women and
(ii) impact on the average skill of working women.

The latter can itself be decomposed into:
(iia) a movement along the "control function" or "selection rule" that represents how the average woman worker's skill is different from the average woman's at each employment rate, and
(iib) a shift of control function.

It has been said that inequality would reduce women's relative wages, and therefore the trends shown above are "puzzling." We agree that effect (i) goes in this direction as long as the average skill among working women is less than the average among working men. As a larger fraction of women are in the workforce over time, the effect (iia) reinforces this only if the selection rule is positive: i.e., if the less-skilled women had been out of the workforce and moving them in lowers the average.

But Mulligan and Rubinstein (2008) argue that effect (iib) is the dominant one, and goes in the direction of raising the average skill among working women. The directions of these two skill effects are proven below.

Setup

```
Get["http://economicreasoning.com"]
```

Proof & Logic Tools 6.2

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Type ERCommands for a list of commands in the package.

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Employment rate *p*

$$\texttt{p}[\sigma\_, \texttt{meangap}\_] = \int_{-\infty}^{\infty}\int_{-\infty}^{\sigma\,\texttt{h}-\texttt{meangap}} \texttt{f}[\texttt{h}, \texttt{r}]\, d\texttt{r}\, d\texttt{h};$$

Average human capital *H*

$$\texttt{H}[\sigma\_, \texttt{meangap}\_] = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\sigma\,\texttt{h}-\texttt{meangap}} \texttt{h}\,\texttt{f}[\texttt{h}, \texttt{r}]\, d\texttt{r}\, d\texttt{h}}{\texttt{p}[\sigma, \texttt{meangap}]};$$

Experiment: increase *σ* but keep employment constant by shifting labor supply with μ_R

$$\texttt{DefineExperiment} = \left\{\frac{d\texttt{p}[\sigma, \mu\texttt{R} - \mu\texttt{w}]}{d\texttt{x}} == \frac{d\mu\texttt{w}}{d\texttt{x}} == 0,\ \frac{d\sigma}{d\texttt{x}} > 0\right\};$$

Useful assumptions

$$\texttt{AllPeopleOfftheMargin} = \int_{-\infty}^{\infty} \texttt{f}[\texttt{h}, \mu\texttt{w} - \mu\texttt{R} + \texttt{h}\,\sigma]\, d\texttt{h} == \int_{-\infty}^{\infty} \texttt{h}\,\texttt{f}[\texttt{h}, \mu\texttt{w} - \mu\texttt{R} + \texttt{h}\,\sigma]\, d\texttt{h} == \int_{-\infty}^{\infty} \texttt{h}^2\,\texttt{f}[\texttt{h}, \mu\texttt{w} - \mu\texttt{R} + \texttt{h}\,\sigma]\, d\texttt{h} == 0;$$

$$\texttt{PeopleOntheMargin} = \texttt{Not}[\texttt{AllPeopleOfftheMargin}];$$

$$\texttt{PositiveEmployment} = \texttt{p}[\sigma, \mu\texttt{R} - \mu\texttt{w}] > 0;$$

$$\texttt{InequalityOntheMargin} = \frac{\int_{-\infty}^{\infty} \texttt{h}^2\,\texttt{f}[\texttt{h}, \mu\texttt{w} - \mu\texttt{R} + \texttt{h}\,\sigma]\, d\texttt{h}}{\int_{-\infty}^{\infty} \texttt{f}[\texttt{h}, \mu\texttt{w} - \mu\texttt{R} + \texttt{h}\,\sigma]\, d\texttt{h}} > \left(\frac{\int_{-\infty}^{\infty} \texttt{h}\,\texttt{f}[\texttt{h}, \mu\texttt{w} - \mu\texttt{R} + \texttt{h}\,\sigma]\, d\texttt{h}}{\int_{-\infty}^{\infty} \texttt{f}[\texttt{h}, \mu\texttt{w} - \mu\texttt{R} + \texttt{h}\,\sigma]\, d\texttt{h}}\right)^2 \land \texttt{PeopleOntheMargin};$$

$$\texttt{ProbabilityProperties} = \left\{\int_{-\infty}^{\infty} \texttt{f}[\texttt{h}, \mu\texttt{w} - \mu\texttt{R} + \texttt{h}\,\sigma]\, d\texttt{h} \ge 0,\ \int_{-\infty}^{\infty} \texttt{h}^2\,\texttt{f}[\texttt{h}, \mu\texttt{w} - \mu\texttt{R} + \texttt{h}\,\sigma]\, d\texttt{h} \ge 0,\ \frac{\int_{-\infty}^{\infty} \texttt{h}^2\,\texttt{f}[\texttt{h}, \mu\texttt{w} - \mu\texttt{R} + \texttt{h}\,\sigma]\, d\texttt{h}}{\int_{-\infty}^{\infty} \texttt{f}[\texttt{h}, \mu\texttt{w} - \mu\texttt{R} + \texttt{h}\,\sigma]\, d\texttt{h}} \ge \left(\frac{\int_{-\infty}^{\infty} \texttt{h}\,\texttt{f}[\texttt{h}, \mu\texttt{w} - \mu\texttt{R} + \texttt{h}\,\sigma]\, d\texttt{h}}{\int_{-\infty}^{\infty} \texttt{f}[\texttt{h}, \mu\texttt{w} - \mu\texttt{R} + \texttt{h}\,\sigma]\, d\texttt{h}}\right)^2 \lor \texttt{Not@PeopleOntheMargin}\right\};$$

Useful definitions

$$\texttt{SkillImpact} = \frac{d\texttt{H}[\sigma, \mu\texttt{R} - \mu\texttt{w}]}{d\texttt{x}};$$

$$\texttt{AverageMinusMarginalSkill} = \texttt{H}[\sigma, \mu\texttt{R} - \mu\texttt{w}] - \frac{\int_{-\infty}^{\infty} \texttt{h}\,\texttt{f}[\texttt{h}, \mu\texttt{w} - \mu\texttt{R} + \texttt{h}\,\sigma]\, d\texttt{h}}{\int_{-\infty}^{\infty} \texttt{f}[\texttt{h}, \mu\texttt{w} - \mu\texttt{R} + \texttt{h}\,\sigma]\, d\texttt{h}};$$

```
rtext = {LogicalExpand@Not@PeopleOntheMargin → "Nobody on the margin",
   LogicalExpand@Not@InequalityOntheMargin → "Equality on the margin"};
```

Results for Employment Constant

```
TheoryGuru[{DefineExperiment, ProbabilityProperties,
   PositiveEmployment,
   PeopleOntheMargin, InequalityOntheMargin},
  SkillImpact > 0]
```

True

```
TheoryGuru[{DefineExperiment, ProbabilityProperties,
   PositiveEmployment,
   Not@PeopleOntheMargin ∨ Not@InequalityOntheMargin},
  SkillImpact == 0]
```

True

```
TheoryOverlap[ProbabilityProperties,
   Not@InequalityOntheMargin,
   Not@PeopleOntheMargin] /. rtext
```

Equality on the margin is necessary but not sufficient for Nobody on the margin

Note that TheoryGuru automatically recognizes the integrals as real numbers

```
TheorySpace@@MostRecentGuruTheory//OtherTools`TFPrintL;
```

$$\left\{\frac{d\mu R}{dx}, \frac{d\mu w}{dx}, \frac{d\sigma}{dx}, \int_{-\infty}^{\infty} f(h, h\,\sigma - \mu R + \mu w)\,dh, \int_{-\infty}^{\infty} h\, f(h, h\,\sigma - \mu R + \mu w)\,dh, \int_{-\infty}^{\infty} h^2\, f(h, h\,\sigma - \mu R + \mu w)\,dh, \int_{-\infty}^{\infty}\int_{-\infty}^{h\,\sigma-\mu R+\mu w} f(h, r)\,dr\,dh, \int_{-\infty}^{\infty}\int_{-\infty}^{h\,\sigma-\mu R+\mu w} h\, f(h, r)\,dr\,dh\right\}$$

Note that μ_w could offset σ instead of, or in addition to, μ_R

```
DefineExperiment[[1, {1, 3}]]
Last@DefineExperiment
```

$$\frac{d\int_{-\infty}^{\infty}\int_{-\infty}^{-\mu R+\mu w+h\,\sigma} f[h, r]\,dr\,dh}{dx} == 0$$

$$\frac{d\sigma}{dx} > 0$$

```
TheoryGuru[{DefineExperiment[[1, {1, 3}]], Last@DefineExperiment,
  ProbabilityProperties, PositiveEmployment,
  PeopleOntheMargin, InequalityOntheMargin},
 SkillImpact > 0]
```

```
True
```

Variable interpretations

Movement Along the Control Function

$$\text{MoveAlongControlFunction} = \left\{\frac{d\sigma}{dx} == \frac{d\mu w}{dx} == 0,\ \frac{dp[\sigma, \mu R - \mu w]}{dx} > 0\right\};$$

The control function can slope up or down, depending on the sign of the selection

```
TheoryGuru[{MoveAlongControlFunction,
  PositiveEmployment, PeopleOntheMargin},

 SameSign[AverageMinusMarginalSkill,
  -SkillImpact]]
```

```
True
```