

Technological Progress

TheoryGuru applied to Chicago Price Theory

(c) Copyright 2019 by JMJ Economics

Setup

Load Economicreasoning package only if it is not already loaded

```
In[ ]:= If[Length@Names["PLTools`*"] < 10,  
  Get["http://economicreasoning.com"]]
```

Proof & Logic Tools 6.3

(c) Copyright 2016, 2017, 2018, 2019 by JMJ Economics

Type ERCommands for a list of commands in the package.

Introduction to Automated Economic Reasoning		
Tutorials:	Entering calculus	General Mathematica tips
Get started	Load extras	Browse examples

Setup

```
In[ ]:= competitivefactormarkets = { $\Delta S_L - \Delta S_k == \Delta W + \Delta L - \Delta R - \Delta k$ ,  
   $\Delta TFP == \Delta Y - (S_L \Delta L + S_k \Delta k)$ ,  
   $\Delta TFP == S_L \Delta W + S_k \Delta R$ ,  
   $\Delta B == \Delta W - \Delta R - \frac{1}{\sigma} (\Delta k - \Delta L)$  (* also embeds homothetic production *)};
```

```
In[ ]:= longrun =  $\Delta R == 0$ ;
```

```
In[ ]:= shortrun = { $\Delta L == 0$ ,  $\Delta k == 0$ };
```

```
In[ ]:= unbiased =  $\Delta B == 0$ ;
```

```
In[ ]:= capitalbiased =  $\Delta B < 0$ ;
```

```
In[ ]:= constantshare = { $\Delta S_L == 0$ ,  $\Delta S_k == 0$ };
```

Growth Results

Long-run link between TFP and the real wage

In[]:= TheoryGuru[{competitivefactormarkets[[3]], longrun}, ΔW == $\frac{\Delta TFP}{S_L}$]

Out[]:= True

Short-run link between TFP and the real wage

In[]:= TheoryGuru[{competitivefactormarkets[[3 ;; 4]], S_L + S_k == 1, shortrun}, ΔW == ΔTFP + S_k ΔB]

Out[]:= True

Sufficient conditions for signing technological bias

In[]:= TheoryGuru[{competitivefactormarkets[[4]],
ΔW - ΔR < 0, Δk - ΔL > 0, σ > 0},
ΔB < 0]

Out[]:= True

Share changes and factor prices

General case

In[]:= TheoryGuru[competitivefactormarkets[[{1, -1}]],
ΔS_L - ΔS_k == $\left(\frac{1}{\sigma} - 1\right) (\Delta k - \Delta L) + \Delta B$]

Out[]:= True

Short run

In[]:= TheoryGuru[{competitivefactormarkets[[1]], shortrun}, ΔS_L - ΔS_k == ΔW - ΔR]

Out[]:= True

Long run with constant shares

In[]:= TheoryGuru[{Most@competitivefactormarkets, constantshare, S_L + S_k == 1, longrun},
Δk == ΔY && ΔW == $\frac{\Delta TFP}{S_L}$]

Out[]:= True

Wage changes decomposed into TFP, capital deepening, and technological bias

```
In[*]:= TheoryGuru[{competitivefactormarkets[[3 ;; 4]], SL + Sk == 1},
  ΔW == ΔTFP + Sk (  $\frac{1}{\sigma}$  (Δk - ΔL) + ΔB )]
```

```
Out[*]:= True
```

Long-run capital deepening

```
In[*]:= TheoryGuru[
  {competitivefactormarkets[[4]], ΔW > 0, σ ≥ 1, unbiased || capitalbiased, longrun},
  Δk - ΔL == σ (ΔW - ΔB) > 0]
```

```
Out[*]:= True
```

```
In[*]:= TheoryGuru[{competitivefactormarkets[[3 ;; 4]],
  ΔW > 0, σ ≥ 1, unbiased || capitalbiased, longrun},
  Δk - ΔL == σ (  $\frac{\Delta TFP}{S_L}$  - ΔB ) > 0]
```

```
Out[*]:= True
```

Variable interpretations

Capital-tax Results

Additional setup

```
In[*]:= longrunntax = {ΔR + Δ[1 - τ] == 0, ΔTFP == 0, ΔB == 0};
```

```
In[*]:= taxincrease = {0 < τ < 1, Δτ > 0, τ Δτ + (1 - τ) Δ[1 - τ] == 0};
```

```
In[*]:= sharerestrictions = {Sk + SL == 1, 0 < SL < 1, σ > 0};
```

```
In[*]:= fixedlabor = ΔL == 0;
```

Results

```
In[*]:= TheoryGuru[{competitivefactormarkets[[3]], Most@longrunntax},
  ΔW ==  $\frac{S_k}{S_L}$  Δ[1 - τ]]
```

```
Out[*]:= True
```

In[]:= TheoryGuru[{taxincrease, sharerestrictions},

$$\frac{S_k}{S_L} \Delta[1 - \tau] < 0 \wedge \sigma \frac{\Delta[1 - \tau]}{S_L} < 0 \wedge \frac{S_k}{S_L} \sigma \Delta[1 - \tau] < 0$$

Out[]:= True

In[]:= TheoryGuru[{competitivefactormarkets[[3 ;; 4]], S_k + S_L == 1, longruntax},

$$\Delta k - \Delta L == \sigma \frac{\Delta[1 - \tau]}{S_L}$$

Out[]:= True

In[]:= TheoryGuru[{competitivefactormarkets[[2 ;; 4]], S_k + S_L == 1, longruntax},

$$\Delta Y - \Delta L == \frac{S_k}{S_L} \sigma \Delta[1 - \tau]$$

Out[]:= True

In[]:= TheoryGuru[{competitivefactormarkets[[2 ;; 4]], unbiased, sharerestrictions,
 taxincrease, longruntax},

$$\Delta W < 0 \ \&\& \ \Delta k - \Delta L < 0 \ \&\& \ \Delta Y - \Delta L < 0$$

Out[]:= True

In[]:= TheoryGuru[{competitivefactormarkets[[3 ;; 4]], sharerestrictions,
 fixedlabor, taxincrease, longruntax},

$$\tau S_k (\Delta \tau + \Delta R + \Delta k) + S_L (\Delta W + \Delta L) < 0$$

Out[]:= True