Risk aversion in vector mode

TheoryGuru applications

Load Economicreasoning package only if it is not already loaded

```
If[Length@Names["PLTools`*"] < 10,
 Get["http://economicreasoning.com"]]
```

Clear symbols that are about to be used

```
Remove[p, u, w, ϵ, x, c, R]
```

Notes

ϵ, u', and u'' are each automatically recognized as vectors with length equal to the number of possible investment outcomes. That number must not be less than one but is otherwise arbitrary.

In the Wolfram Language, x.y refers to the tensor DOT PRODUCT, NOT scalar multiplication. For TheoryGuru purposes, tensor means vector, so that the result of x.y is a scalar.

Setup

Take wealth w and allocate x of it to a risky activity that has gross return ϵ.

$$\text{optimum} = \frac{\partial\,(p.u(w+(\epsilon-1)\,x))}{\partial\,x} == 0$$

```
p.((-1+ϵ) u'[w+x (-1+ϵ)]) == 0
```

```
NetReturnisNotAlwaysZero = p.((ϵ-1)^2 u'[w+x (-1+ϵ)]) > 0;
```

Vary wealth w holding ϵ and p constant.

```
ComparativeStatic := Dt[optimum, w] /. (Dt[ϵ, w] | Dt[p, w]._) → 0
```

Possible restrictions on risk aversion

```
AssumingConstantARA = u'[c_] → u''[c] / R;

AssumingConstantRRA = u'[c_] → c u''[c] / R;
```

Results

With constant Absolute Risk Aversion, the optimal risk investment is independent of wealth

```
TheoryGuru[{optimum, ComparativeStatic, NetReturnisNotAlwaysZero,
   R > 0, ProbabilityVector@p, ϵ.ϵ > 0, u'[w + x (-1 + ϵ)].u'[w + x (-1 + ϵ)] > 0} /.
  AssumingConstantARA,
 dx/dw == 0]
```

```
True
```

```
TheorySpace[]
```

```
Using MostRecentTheory.
{R, 1.1, 1.u''[w + x (-1 + ϵ)], ϵ.ϵ, ϵ.ϵ^2, ϵ.u''[w + x (-1 + ϵ)], ϵ^2.ϵ^2,
 ϵ^2.u''[w + x (-1 + ϵ)], u''[w + x (-1 + ϵ)].u''[w + x (-1 + ϵ)], Dt[x, w]}
1, ϵ, ϵ^2, u''[w + x (-1 + ϵ)] are interpreted as vectors.
```

With constant Relative Risk Aversion, the optimal risk investment *share* is independent of wealth

```
optimum = ∂(p.u(w + (ϵ - 1) s w))/∂s == 0;

SecondOrderCondition = ∂^2(p.u(w + (ϵ - 1) s w))/∂s^2 < 0;

TheoryGuru[{optimum, ComparativeStatic, SecondOrderCondition,
   ProbabilityVector@p, w > 0, 0 ≤ s ≤ 1, R > 0} /. AssumingConstantRRA,
 ds/dw == 0]
```

```
True
```

TheorySpace[]

Using MostRecentTheory.

$\{R, s, w, 1.1, 1.u''[w + s\,w\,(-1+\epsilon)], \epsilon.\epsilon, \epsilon.\epsilon^2, \epsilon.u''[w + s\,w\,(-1+\epsilon)], \epsilon^2.\epsilon^2, \epsilon^2.u''[w + s\,w\,(-1+\epsilon)], u''[w + s\,w\,(-1+\epsilon)].u''[w + s\,w\,(-1+\epsilon)], Dt[s, w]\}$

$1, \epsilon, \epsilon^2, u''[w + s\,w\,(-1+\epsilon)]$ are interpreted as vectors.