A Public-Goods Game

TheoryGuru applications

Load Economicreasoning package only if it is not already loaded

```
If[Length@Names["PLTools`*"] < 10,
 Get["http://economicreasoning.com"]]
```

Load other tools by clicking on extras and/or evaluating below

```
If[Not@MemberQ[$ContextPath, "OtherTools`"],
 Get["http://othertools.economicreasoning.com"]]
```

Notes

n identical consumers have preferences over private and public consumption.

Each make a voluntary contribution to public consumption, taking as given the contributions of the others.

Setup

$$\text{mrs}[\text{private\_}, \text{public\_}] := \frac{u^{(0,1)}[\text{private}, \text{public}]}{u^{(1,0)}[\text{private}, \text{public}]}$$

```
public = n * contrib;

NashEquilibrium = mrs[income - contrib, public] == 1;

StableEquilibrium =
  mrs^(1,0)[income - contrib, public] - n mrs^(0,1)[income - contrib, public] > 0;

Publicisnormal = mrs^(1,0)[income - contrib, public] > 0;

Privateisnormal = mrs^(0,1)[income - contrib, public] < 0;
```

```
QuasilinearPrivate = {mrs^(1,0)[income - contrib, public] == 0,
    u^(1,1)[income - contrib, public] == 0,
    u^(0,2)[income - contrib, public] < 0};

QuasilinearPublic = {mrs^(0,1)[income - contrib, public] == 0,
    u^(1,1)[income - contrib, public] == 0,
    u^(2,0)[income - contrib, public] < 0};

SignConditions = {n ≥ 1, contrib > 0,
    u^(1,0)[income - contrib, public] > 0,
    u^(0,1)[income - contrib, public] > 0,
    QuasiConcaveFunctionQ[u, {income - contrib, public}, strictly → True]};
```

Logical possibilities for income effects and stability

```
TheoryGuru[SignConditions[[3 ;; -1]],
 Publicisnormal || Privateisnormal || QuasilinearPrivate || QuasilinearPublic]
```

```
True
```

```
TheoryGuru[{SignConditions[[1]], Publicisnormal, Privateisnormal},
 StableEquilibrium]
```

```
True
```

```
TheoryGuru[{SignConditions,
   QuasilinearPrivate || QuasilinearPublic},
 StableEquilibrium,
 keepall → True]
```

```
True
```

```
TheoryGuru[{SignConditions,
   Not@StableEquilibrium},
 Not@Publicisnormal || Not@Privateisnormal,
 keepall → True]
```

```
True
```

Comparative statics for number of participants *n*

Formulas

```
TheoryGuru[{Dt[NashEquilibrium, n], Dt[income, n] == 0,
   StableEquilibrium},
  Dt[contrib, n] == (contrib mrs^(0,1)[income - contrib, public]) /
     (mrs^(1,0)[income - contrib, public] - n mrs^(0,1)[income - contrib, public]) &&
   Dt[public, n] == (contrib mrs^(1,0)[income - contrib, public]) /
     (mrs^(1,0)[income - contrib, public] - n mrs^(0,1)[income - contrib, public])]
```

True

Quasilinear special case

```
TheoryGuru[{Dt[NashEquilibrium, n], Dt[income, n] == 0,
   QuasilinearPrivate, SignConditions},
  Dt[contrib, n] < 0 &&
   Dt[public, n] == 0]
```

True

The sign of the total-contribution impact is the same sign as income effects on public-good demand

```
TheoryOverlap[{Dt[NashEquilibrium, n], Dt[income, n] == 0,
   SignConditions, StableEquilibrium},
  Publicisnormal,
  Dt[public, n] > 0]
```

$$\left\{\frac{\frac{\partial^2 u(\text{income-contrib},\text{contrib}\,n)}{\partial \text{income-contrib}\,\partial \text{contrib}\,n}}{\frac{\partial u(\text{income-contrib},\text{contrib}\,n)}{\partial \text{income-contrib}}} - \left(\frac{\partial u(\text{income-contrib},\text{contrib}\,n)}{\partial \text{contrib}\,n}\,\frac{\partial^2 u(\text{income-contrib},\text{contrib}\,n)}{\partial (\text{income-contrib})^2}\right) \Big/ \left(\frac{\partial u(\text{income-contrib},\text{contrib}\,n)}{\partial \text{income-contrib}}\right)^2 > 0,\ n\,\frac{\text{d contrib}}{\text{d}n} + \text{contrib} > 0\right\}$$ are equivalent

```
TheoryGuru[{Dt[NashEquilibrium, n], Dt[income, n] == 0,
   SignConditions, StableEquilibrium},
  SameSign[Dt[public, n],
   mrs^(1,0)[income - contrib, public]]]
```

True

```
Column[MostRecentAssumption,Spacings→1]
```

$$\left(\left(\text{contrib}+n\,\text{Dt}[\text{contrib},\,n]\right)u^{(0,2)}[-\text{contrib}+\text{income},\,\text{contrib}\,n]+\left(-\text{Dt}[\text{contrib},\,n]+\text{Dt}[\text{income},\,n]\right)u^{(1,1)}[-\text{contrib}+\text{income},\,\text{contrib}\,n]\right)/u^{(1,0)}[-\text{contrib}+\text{income},\,\text{contrib}\,n]-\left(u^{(0,1)}[-\text{contrib}+\text{income},\,\text{contrib}\,n]\left(\left(\text{contrib}+n\,\text{Dt}[\text{contrib},\,n]\right)u^{(1,1)}[-\text{contrib}+\text{income},\,\text{contrib}\,n]+\left(-\text{Dt}[\text{contrib},\,n]+\text{Dt}[\text{income},\,n]\right)u^{(2,0)}[-\text{contrib}+\text{income},\,\text{contrib}\,n]\right)\right)/u^{(1,0)}[-\text{contrib}+\text{income},\,\text{contrib}\,n]^2==0$$

$$\text{Dt}[\text{income},\,n]==0$$

$$n\geq 1$$

$$\text{contrib}>0$$

$$u^{(1,0)}[-\text{contrib}+\text{income},\,\text{contrib}\,n]>0$$

$$u^{(0,1)}[-\text{contrib}+\text{income},\,\text{contrib}\,n]>0$$

$$u^{(1,0)}[-\text{contrib}+\text{income},\,\text{contrib}\,n]^2>0$$

$$2\,u^{(0,1)}[-\text{contrib}+\text{income},\,\text{contrib}\,n]\,u^{(1,0)}[-\text{contrib}+\text{income},\,\text{contrib}\,n]\,u^{(1,1)}[-\text{contrib}+\text{income},\,\text{contrib}\,n]>u^{(0,2)}[-\text{contrib}+\text{income},\,\text{contrib}\,n]\,u^{(1,0)}[-\text{contrib}+\text{income},\,\text{contrib}\,n]^2+u^{(0,1)}[-\text{contrib}+\text{income},\,\text{contrib}\,n]^2\,u^{(2,0)}[-\text{contrib}+\text{income},\,\text{contrib}\,n]$$

$$\frac{u^{(1,1)}[-\text{contrib}+\text{income},\text{contrib}\,n]}{u^{(1,0)}[-\text{contrib}+\text{income},\text{contrib}\,n]}-n\left(\frac{u^{(0,2)}[-\text{contrib}+\text{income},\text{contrib}\,n]}{u^{(1,0)}[-\text{contrib}+\text{income},\text{contrib}\,n]}-\frac{u^{(0,1)}[-\text{contrib}+\text{income},\text{contrib}\,n]\,u^{(1,1)}[-\text{contrib}+\text{income},\text{contrib}\,n]}{u^{(1,0)}[-\text{contrib}+\text{income},\text{contrib}\,n]^2}\right)-\frac{u^{(0,1)}[-\text{contrib}+\text{income},\text{contrib}\,n]\,u^{(2,0)}[-\text{contrib}+\text{income},\text{contrib}\,n]}{u^{(1,0)}[-\text{contrib}+\text{income},\text{contrib}\,n]^2}>0$$

The sign of the per-capita-contribution impact is the opposite sign as income effects on consumption demand

```
TheoryOverlap[{Dt[NashEquilibrium, n], Dt[income, n] == 0,
  SignConditions, StableEquilibrium},
 Privateisnormal,
 Dt[contrib, n] < 0]
```

$$\left\{\frac{\frac{\partial^2 u(\text{income}-\text{contrib},\text{contrib}\,n)}{\partial(\text{contrib}\,n)^2}}{\frac{\partial u(\text{income}-\text{contrib},\text{contrib}\,n)}{\partial\text{income}-\text{contrib}}}-\left(\frac{\partial u(\text{income}-\text{contrib},\text{contrib}\,n)}{\partial\text{contrib}\,n}\,\frac{\partial^2 u(\text{income}-\text{contrib},\text{contrib}\,n)}{\partial\text{income}-\text{contrib}\,\partial\text{contrib}\,n}\right)\Big/\left(\frac{\partial u(\text{income}-\text{contrib},\text{contrib}\,n)}{\partial\text{income}-\text{contrib}}\right)^2<0,\ \frac{d\text{contrib}}{dn}<0\right\}\ \text{are equivalent}$$

```
TheoryGuru[{Dt[NashEquilibrium, n], Dt[income, n] == 0,
  SignConditions, StableEquilibrium},
 SameSign[Dt[contrib, n],
  mrs^(0,1)[income - contrib, public]]]
```

True

Variable interpretations